

# Shadowfax: Combiners for Deniability

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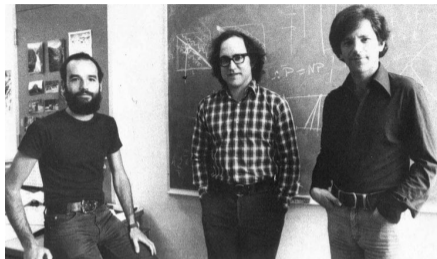
Cryptographic Applications Workshop at EUROCRYPT 2025, Madrid, Spain.

MAX PLANCK INSTITUTE  
FOR SECURITY AND PRIVACY

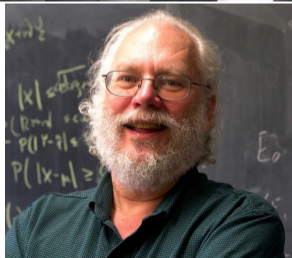
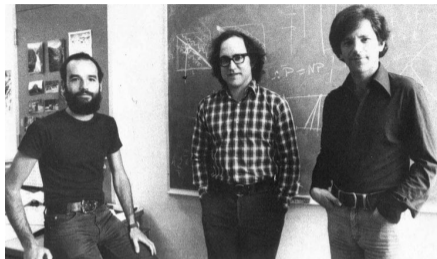


**CASA**  
CYBER SECURITY IN THE AGE  
OF LARGE-SCALE ADVERSARIES

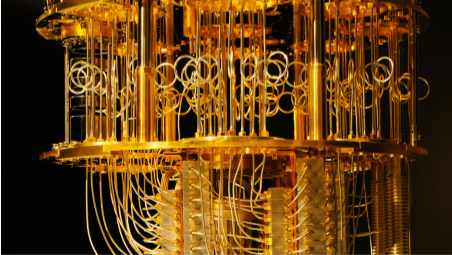
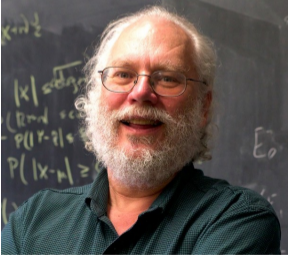
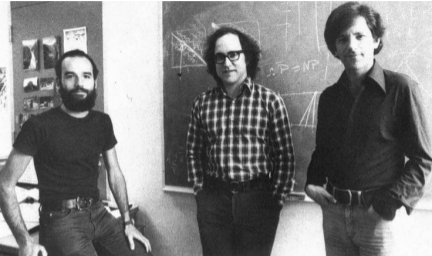
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- ▶ NIST PQC standards for KEMs and Sig schemes.
- ▶ Protocols are being adapted to be post-quantum secure.
  - ▶ X3DH → PQXDH (deployed in Signal) [MP16, KS24].
  - ▶ Apple's iMessage → PQ3 [App24].
  - ▶ TLS → [BCNS15], [PST20], [BBCT22], [Lan16], [Lan18], [KV19], [WR19], ....
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- ▶ **Classical setting:** X3DH provides deniability via implicit authentication.
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## A Deniability Analysis of Signal’s Initial Handshake PQXDH

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### ABSTRACT

Many use messaging apps such as Signal to exercise their right to private communication. To cope with the advent of quantum computing, Signal employs a new initial handshake protocol called PQXDH for post-quantum confidentiality, yet keeps guarantees of authenticity and deniability classical. Compared to its predecessor X3DH, PQXDH includes a KEM encapsulation and a signature on the ephemeral key. In this work we show that PQXDH does not meet

scenario without computers and smartphones present, where Alice and Bob have a conversation. Later, Alice tells Charlie she talked to Bob, and Bob tells Charlie he never talked to Alice. Charlie does not know whose word to trust; hence, Bob can *deny* his conversation with Alice since their conversation did not leave any evidence behind. For sensitive topics such as medical conditions, sexuality, or coordinating a protest against an oppressive regime, this type of privacy guarantee can be not just helpful but even vital.

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*Q: “Can combiners preserve deniability in a post-quantum setting at minimal additional cost?”*

# OUTLINE

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- ▶ Combiners: Two Classes
- ▶ Case Study: Authenticated KEM
- ▶ Black-box Construction: Problems
- ▶ SHADOWFAX
  - ▶ Construction
  - ▶ Instantiation
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# COMBINERS

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- ▶ Construct a secure scheme  $\Pi$  from two schemes  $\Pi_1$  and  $\Pi_2$ .
- ▶ Security is *binary*: either secure or insecure.
- ▶ Goal:  $\Pi_1 \vee \Pi_2 \implies \Pi$ .
- ▶ Examples of  $\Pi$ :
  - ▶  $\text{PRG} := \text{PRG}_1(s_1) \oplus \text{PRG}_2(s_2)$
  - ▶ KEM with  $k = H(k_1, k_2, c_1, c_2)$  (RO model)
  - ▶ KEM with  $k = \text{PRF}(k_1, c_1 || c_2) \oplus \text{PRF}(k_2, c_1 || c_2)$  (standard model [GHP18])

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- ▶ Build  $\Pi$  from classical ( $\Pi_{\text{pre-Q}}$ ) and post-quantum ( $\Pi_{\text{post-Q}}$ ) components.

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A: Classical implementations are mature; PQC is harder to implement. (code audits)

A: PQC schemes may lack formal proofs. (formal verification)

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- Quantum computers become practical (leak classical computers' secrets, etc.)
- Post-quantum assumptions are flawed (LWE, SIDH, ... may not hold)

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Can relax to:  $\text{Assumption}_{\text{pre-Q}} \vee \text{Assumption}_{\text{post-Q}} \implies \Pi$

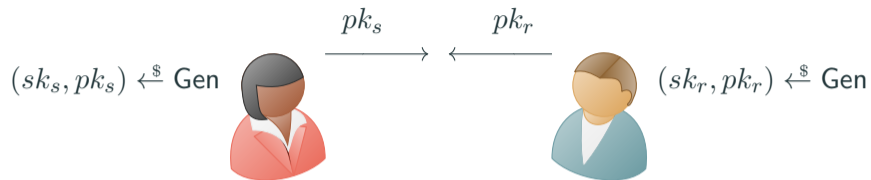
# CASE STUDY: AUTHENTICATED KEM

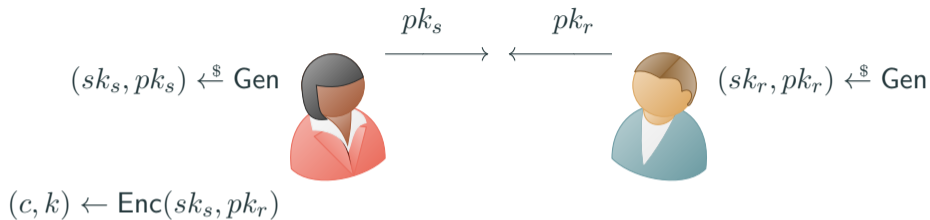
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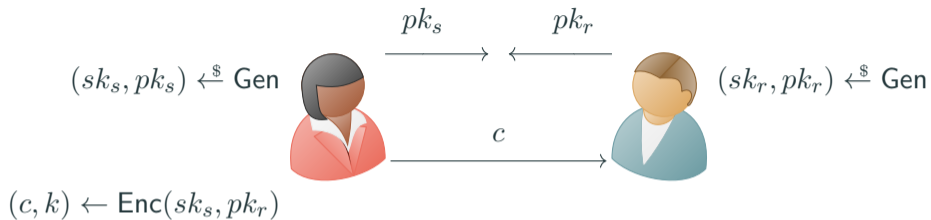


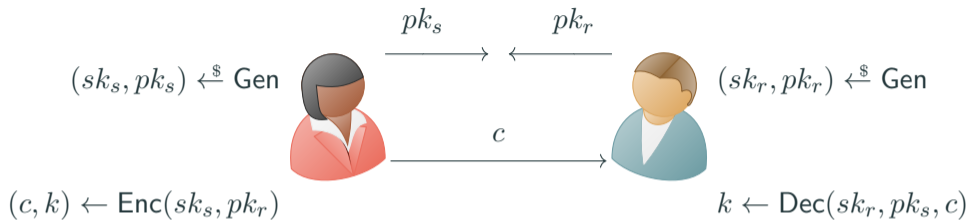
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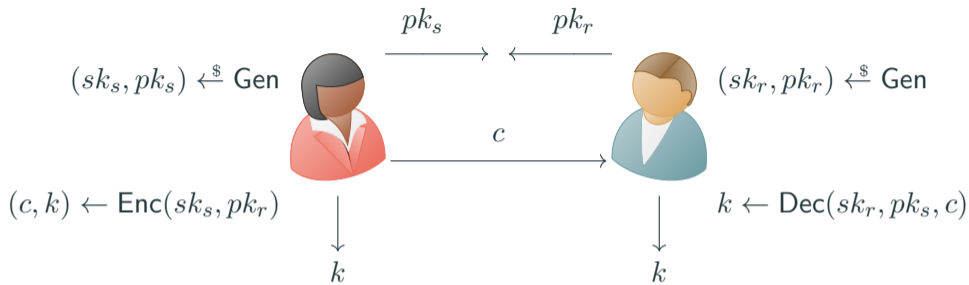
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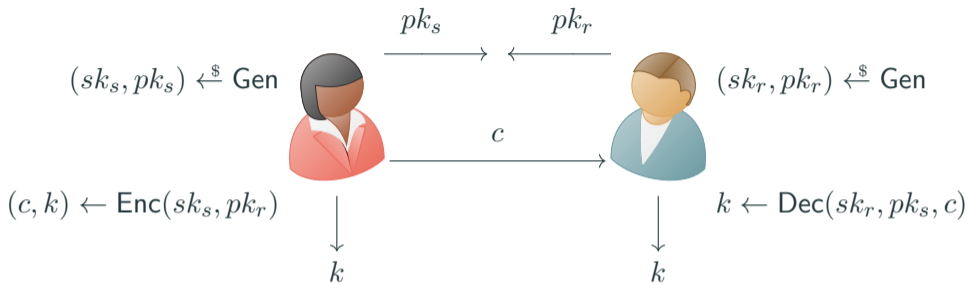




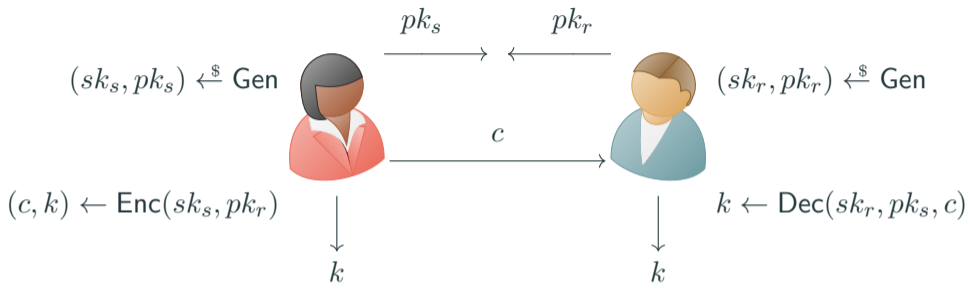




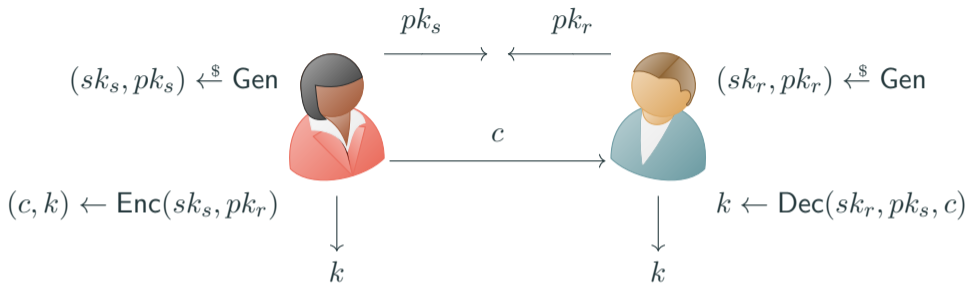




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- ▶ **Authenticity:**  $r$  knows  $s$  sent the ciphertext  $c$ .
- ▶ **Deniability:**  $s$  can deny having sent  $c$  to  $r$ .
  - ▶  $\exists \text{ Sim} : \text{Enc}(sk_s, pk_r) \approx \text{Sim}(\dots)$  (honest/dishonest receivers)

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- ▶ Messaging
  - ▶ Messaging Layer Security (MLS) [BBR<sup>+</sup>23].
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# BLACK-BOX CONSTRUCTION OF A DENIABLE AKEM := [AKEM<sub>1</sub>, AKEM<sub>2</sub>, H]

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$(sk_2, pk_2) \leftarrow^{\$} \text{AKEM}_2.\text{Gen}$

return  $((sk_1, sk_2), (pk_1, pk_2))$

Enc( $sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2)$ )

$(c_1, k_1) \leftarrow^{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$

$(c_2, k_2) \leftarrow^{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$

$c := (c_1, c_2)$

$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$

return  $(c, k)$

Dec( $pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c$ )

parse  $c \rightarrow (c_1, c_2)$

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$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$

**return**  $(c, k)$

Dec( $pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c$ )

parse  $c \rightarrow (c_1, c_2)$

$k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$

$k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$

$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$

**return**  $k$

# BLACK-BOX CONSTRUCTION OF A DENIABLE AKEM := [AKEM<sub>1</sub>, AKEM<sub>2</sub>, H]

Gen

$(sk_1, pk_1) \leftarrow^{\$} \text{AKEM}_1.\text{Gen}$

$(sk_2, pk_2) \leftarrow^{\$} \text{AKEM}_2.\text{Gen}$

**return**  $((sk_1, sk_2), (pk_1, pk_2))$

Enc( $sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2)$ )

$(c_1, k_1) \leftarrow^{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$

$(c_2, k_2) \leftarrow^{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$

$c := (c_1, c_2)$

$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$

**return**  $(c, k)$

Dec( $pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c$ )

**parse**  $c \rightarrow (c_1, c_2)$

$k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$

$k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$

$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$

**return**  $k$

# BLACK-BOX CONSTRUCTION OF A DENIABLE AKEM := [AKEM<sub>1</sub>, AKEM<sub>2</sub>, H]

<b>Gen</b> $(sk_1, pk_1) \xleftarrow{\$} \text{AKEM}_1.\text{Gen}$ $(sk_2, pk_2) \xleftarrow{\$} \text{AKEM}_2.\text{Gen}$ <b>return</b> $((sk_1, sk_2), (pk_1, pk_2))$	<b>Enc</b> $(sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2))$ $(c_1, k_1) \xleftarrow{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$ $(c_2, k_2) \xleftarrow{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$ $c := (c_1, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $(c, k)$	<b>Dec</b> $(pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c)$ <b>parse</b> $c \rightarrow (c_1, c_2)$ $k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$ $k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $k$
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- ▶ **Confidentiality:** follows from either AKEM<sub>1</sub> **or** AKEM<sub>2</sub> ( $\approx$  KEM combiner)
  - ▶ However,  $k$  also hashes sender and receiver public keys to satisfy Ins-CCA.
- ▶ **Authenticity:** follows from either AKEM<sub>1</sub> **or** AKEM<sub>2</sub>.
- ▶ **Deniability:** follows from both AKEM<sub>1</sub> **and** AKEM<sub>2</sub>.

<b>Gen</b> $(sk_1, pk_1) \xleftarrow{\$} \text{AKEM}_1.\text{Gen}$ $(sk_2, pk_2) \xleftarrow{\$} \text{AKEM}_2.\text{Gen}$ <b>return</b> $((sk_1, sk_2), (pk_1, pk_2))$	<b>Enc</b> $(sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2))$ $(c_1, k_1) \xleftarrow{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$ $(c_2, k_2) \xleftarrow{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$ $c := (c_1, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $(c, k)$	<b>Dec</b> $(pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c)$ <b>parse</b> $c \rightarrow (c_1, c_2)$ $k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$ $k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $k$
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<b>Gen</b> $(sk_1, pk_1) \xleftarrow{\$} \text{AKEM}_1.\text{Gen}$ $(sk_2, pk_2) \xleftarrow{\$} \text{AKEM}_2.\text{Gen}$ <b>return</b> $((sk_1, sk_2), (pk_1, pk_2))$	<b>Enc</b> $(sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2))$ $(c_1, k_1) \xleftarrow{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$ $(c_2, k_2) \xleftarrow{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$ $c := (c_1, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $(c, k)$	<b>Dec</b> $(pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c)$ <b>parse</b> $c \rightarrow (c_1, c_2)$ $k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$ $k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$ $k := \text{H}(k_1, k_2, pk_s, pk_r, c)$ <b>return</b> $k$
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Gen	Enc( $sk_s := (sk_1, sk_2), pk_r := (pk_1, pk_2)$ )	Dec( $pk_s := (pk_1, pk_2), sk_r := (sk_1, sk_2), c$ )
$(sk_1, pk_1) \xleftarrow{\$} \text{AKEM}_1.\text{Gen}$	$(c_1, k_1) \xleftarrow{\$} \text{AKEM}_1.\text{Enc}(sk_1, pk_1)$	<b>parse</b> $c \rightarrow (c_1, c_2)$
$(sk_2, pk_2) \xleftarrow{\$} \text{AKEM}_2.\text{Gen}$	$(c_2, k_2) \xleftarrow{\$} \text{AKEM}_2.\text{Enc}(sk_2, pk_2)$	$k_1 \leftarrow \text{AKEM}_1.\text{Dec}(pk_1, sk_1, c_1)$
<b>return</b> $((sk_1, sk_2), (pk_1, pk_2))$	$c := (c_1, c_2)$	$k_2 \leftarrow \text{AKEM}_2.\text{Dec}(pk_2, sk_2, c_2)$
	$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$	$k := \text{H}(k_1, k_2, pk_s, pk_r, c)$
	<b>return</b> $(c, k)$	<b>return</b> $k$

- ▶ **Confidentiality:** follows from either AKEM<sub>1</sub> **or** AKEM<sub>2</sub> ( $\approx$  KEM combiner)
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**Q:** *What does it mean to lose deniability?*

Cannot construct  $\text{Sim} : \text{Enc}(sk_s, pk_r) \approx \text{Sim}(\dots)$

**Q:** *Computationally or Statistically Indistinguishable?*

Comp. deniability:  $\neg \text{Assumption}_{\text{pre-Q}} \vee \neg \text{Assumption}_{\text{post-Q}} \implies \neg \text{Deniability}$

Stat. deniability:  $\neg \text{Assumption}_{\text{pre-Q}} \vee \neg \text{Assumption}_{\text{post-Q}} \not\Rightarrow \neg \text{Deniability}$

**A:** Use statistical deniability. **Not possible** for confidentiality/authenticity!

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SHADOWFAX

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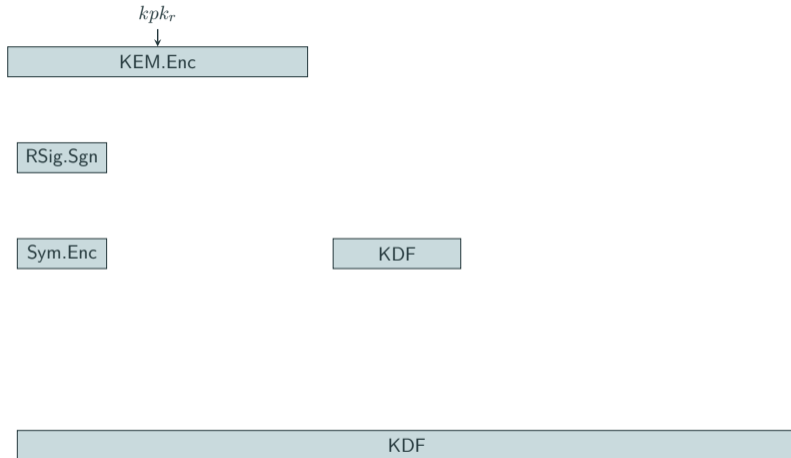
KEM.Enc

RSig.Sgn

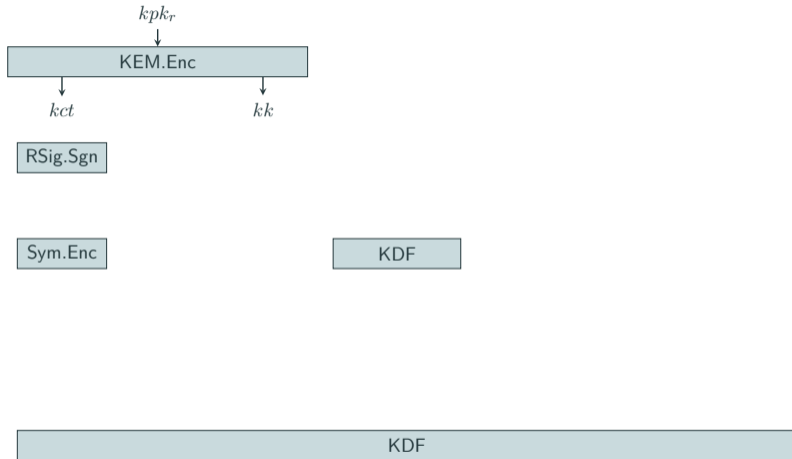
Sym.Enc

KDF

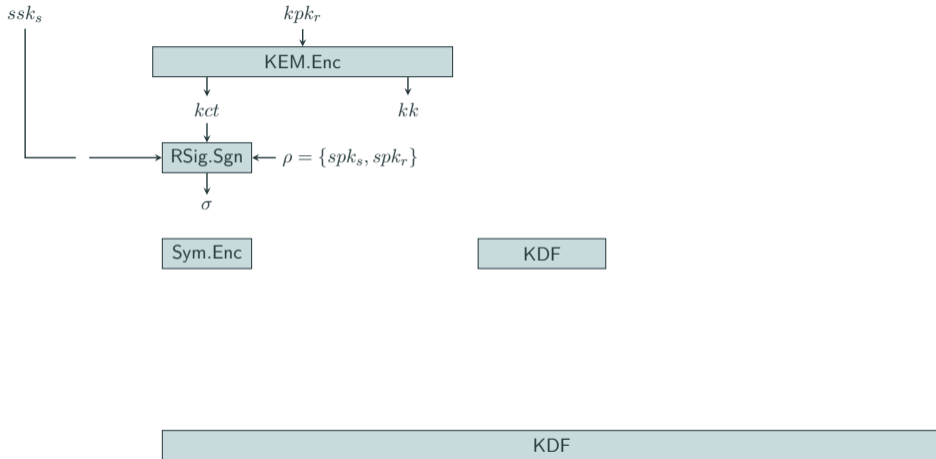
KDF



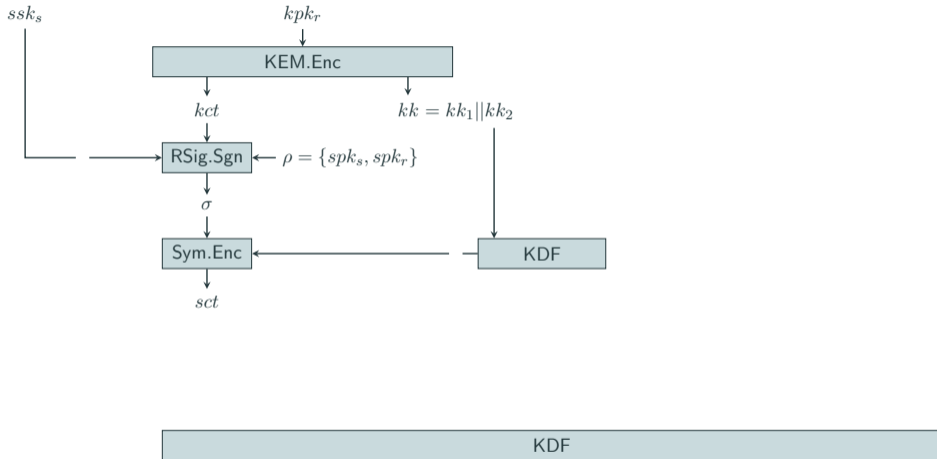
# DENIABLE AKEM := (Gen, Enc, Dec) — SHADOWFAX CONSTRUCTION



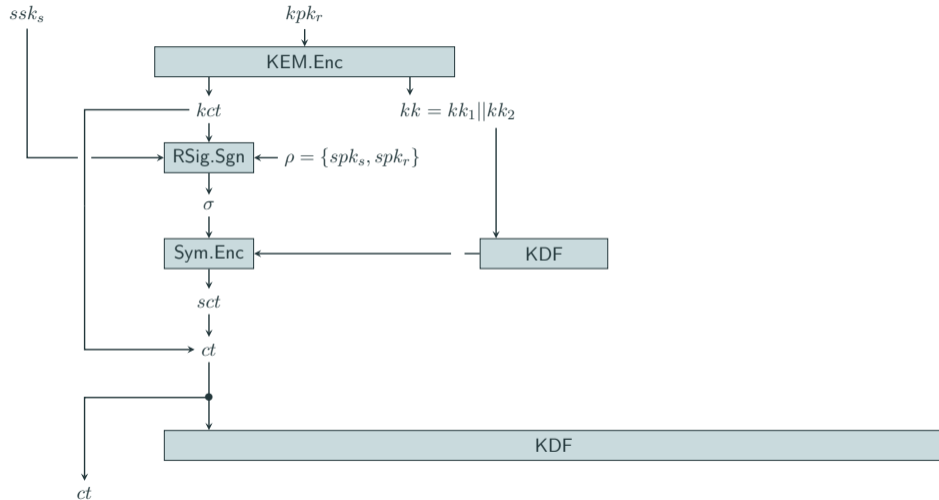
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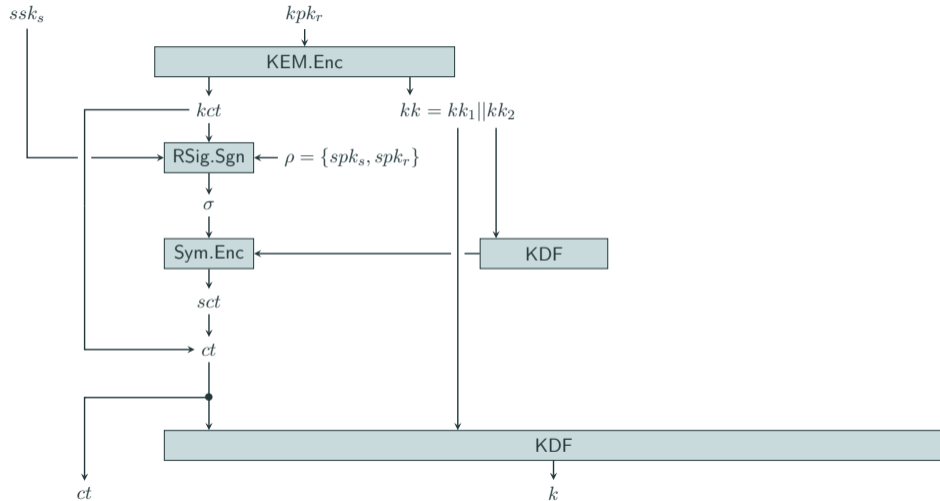
# DENIABLE AKEM := (Gen, Enc, Dec) — SHADOWFAX CONSTRUCTION



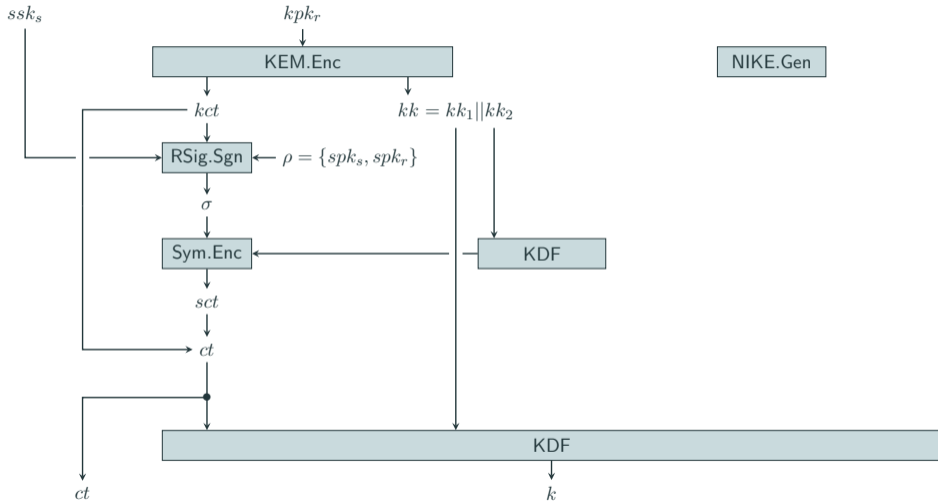
# DENIABLE AKEM := (Gen, Enc, Dec) — SHADOWFAX CONSTRUCTION



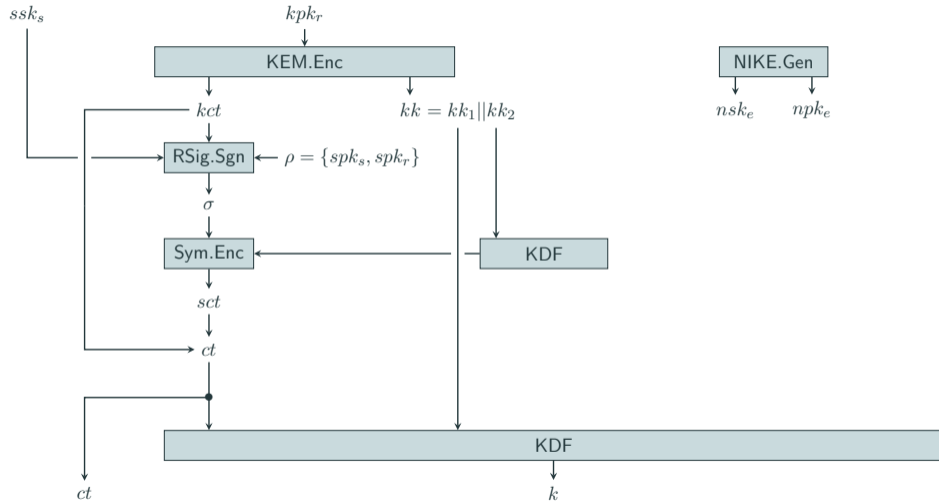
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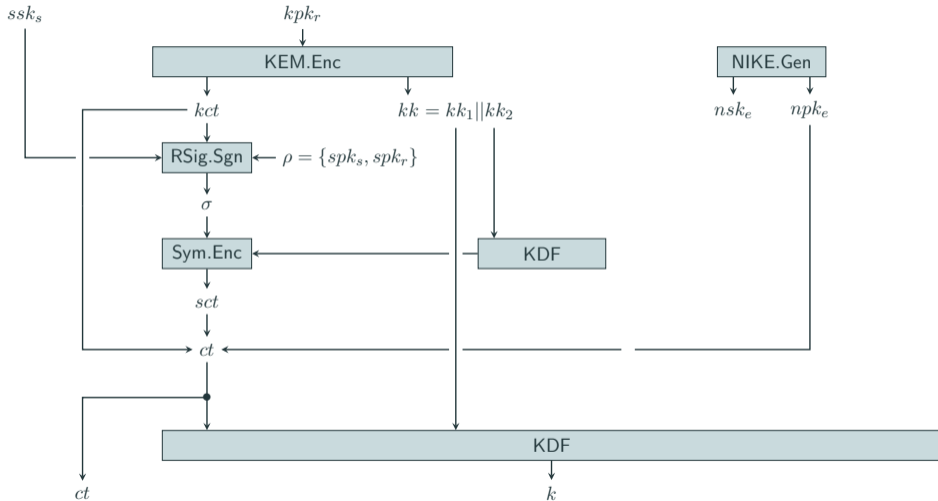
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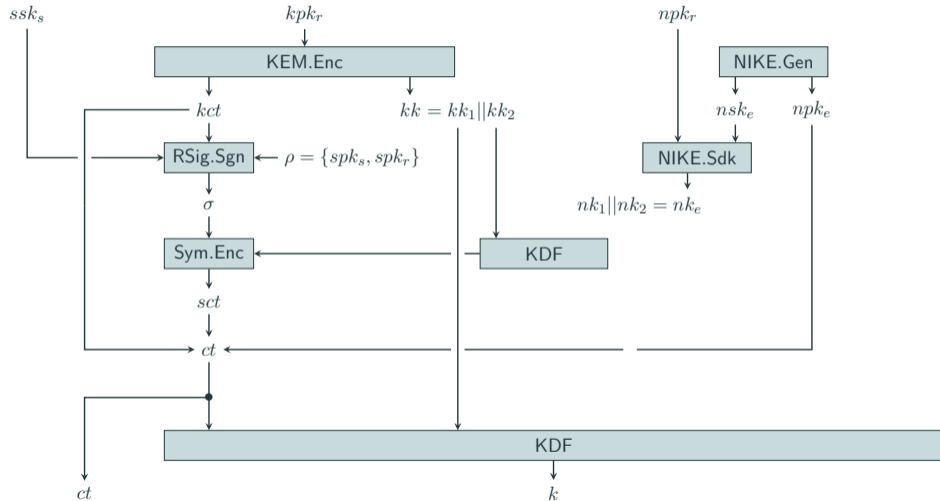
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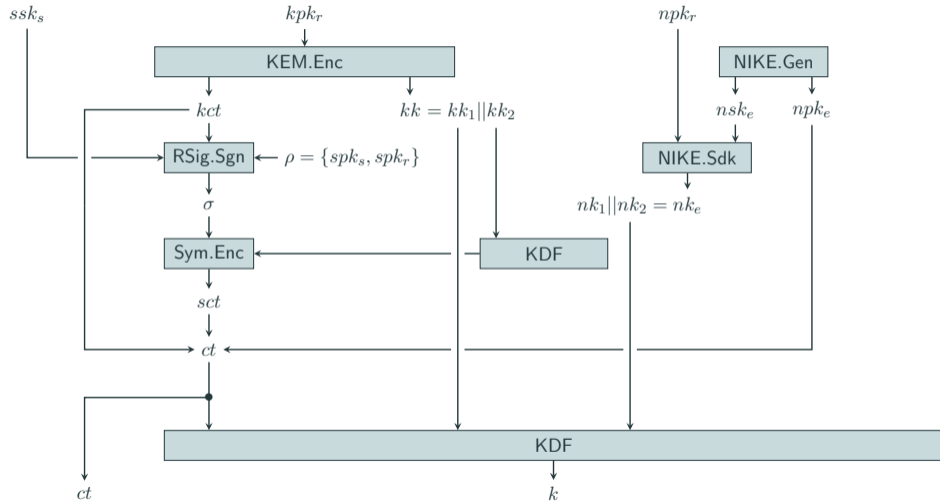
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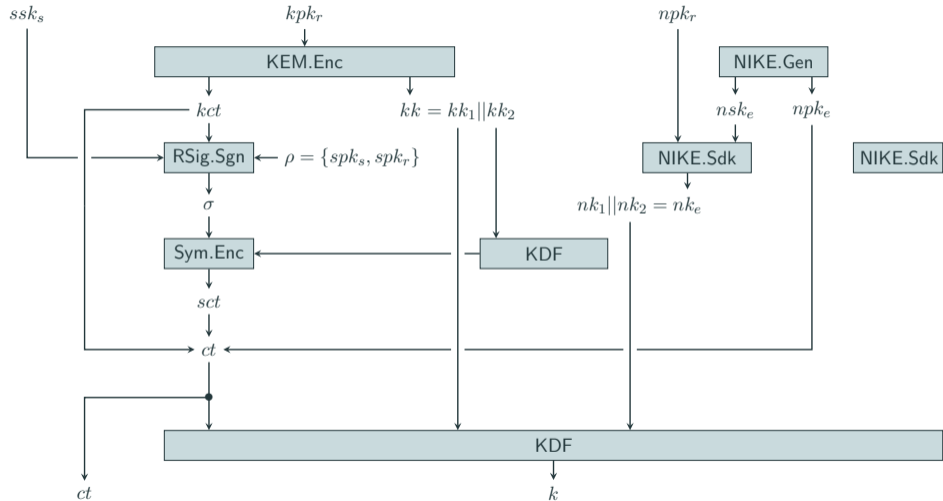
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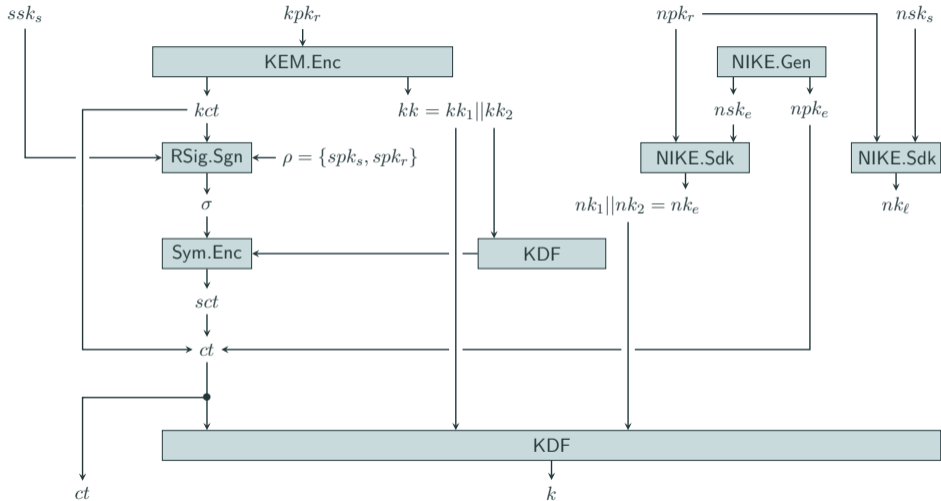
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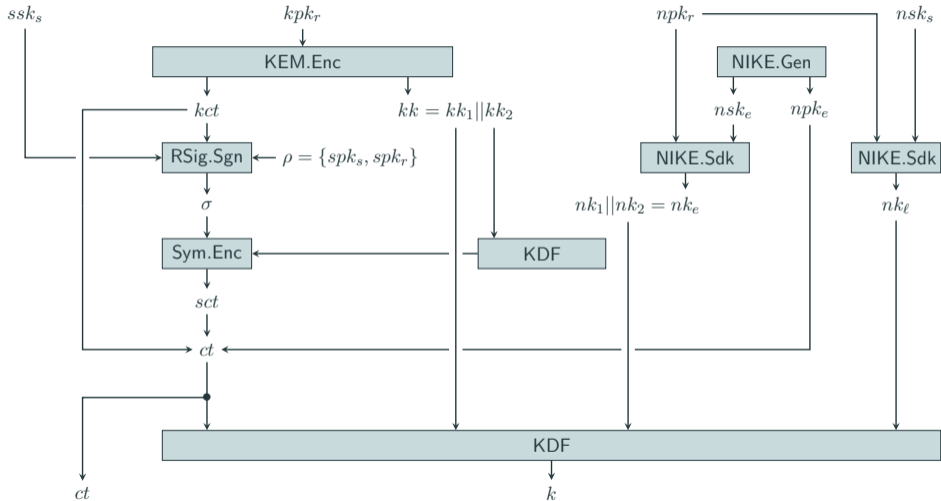
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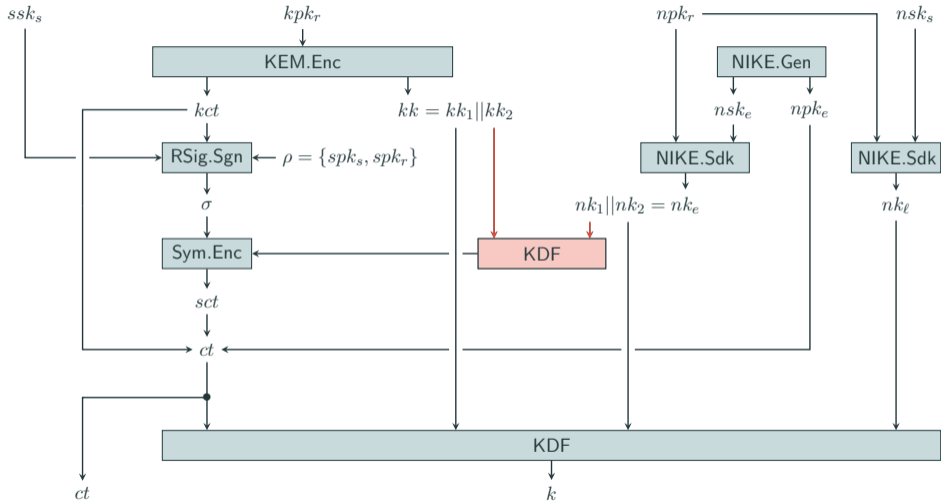
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# INSTANTIATIONS OF PRIMITIVES USED IN SHADOWFAX

Primitive	Scheme (variant)		Assumption	Size (in bytes)		
				$\sigma$	$c$	$pk$
NIKE	x25519 [Ber06]		CDH	—	32	32

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				$\sigma$	$c$	$pk$
NIKE	x25519 [Ber06]		CDH	—	32	32
KEM	bat_257_512 [FKPY22]		NTRU, Ring-LWR	—	473	521

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				$\sigma$	$c$	$pk$
NIKE	X25519 [Ber06]		CDH	—	32	32
KEM	bat_257_512 [FKPY22]		NTRU, Ring-LWR	—	473	521
	ML-KEM-512 [ML-24]		M-LWE	—	768	800

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				$\sigma$	$c$	$pk$
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KEM	bat_257_512 [FKPY22]		NTRU, Ring-LWR	—	473	521
	ML-KEM-512 [ML-24]		M-LWE	—	768	800
RSig	GANDALF[TpdGen, PreSmp] [GJK24]	ANTRAG [ENS <sup>+</sup> 23], MITAKA [EFG <sup>+</sup> 22]	NTRU, Ring-ISIS	1 236	—	896

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Primitive	Scheme (variant)		Assumption	Size (in bytes)		
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		Falcon-512 [PFH <sup>+</sup> 22]		1 276	—	897

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		Falcon-512 [PFH <sup>+</sup> 22]		1 276	—	897

- ▶ We opt for the most compact instantiations.
- ▶ However, also possible to use NIST PQC standards.

Scheme	Instantiation	Deniability	Assumption		Size (in bytes)	
			pre-Q	post-Q	$c$	$pk$
DH-AKEM [ABH <sup>+</sup> 21]	X25519	DR-Den*	✓	✗	32	32

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Deniability properties marked with an “\*” have not been formally proven in the respective work.

Scheme	Instantiation	Deniability	Assumption		Size (in bytes)	
			pre-Q	post-Q	$c$	$pk$
DH-AKEM [ABH <sup>+</sup> 21]	X25519	DR-Den*	✓	✗	32	32
ETStH-AKEM [AJKL23]	bat_257_542 + ANTRAG	✗	✗	✓	1 119	1 417
	ML-KEM-512 + Falcon-512				1 434	1 697

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Scheme	Instantiation	Deniability	Assumption		Size (in bytes)	
			pre-Q	post-Q	$c$	$pk$
DH-AKEM [ABH <sup>+</sup> 21]	X25519	DR-Den*	✓	✗	32	32
ETStH-AKEM [AJKL23]	bat_257_542 + ANTRAG	✗	✗	✓	1 119	1 417
	ML-KEM-512 + Falcon-512				1 434	1 697
NIKE-AKEM [AJKL23]	SWOOSH [GdKQ <sup>+</sup> 24]	DR-Den*	✗	✓	> 221 184	> 221 184

---

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			pre-Q	post-Q	$c$	$pk$
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ETStH-AKEM [AJKL23]	bat_257_542 + ANTRAG	✗	✗	✓	1 119	1 417
	ML-KEM-512 + Falcon-512				1 434	1 697
NIKE-AKEM [AJKL23]	SWOOSH [GdKQ <sup>+</sup> 24]	DR-Den*	✗	✓	> 221 184	> 221 184
FrodoKEX+ [CHN <sup>+</sup> 24]	n/a	DR-Den	✗	✓	72	21 300

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Deniability properties marked with an “\*” have not been formally proven in the respective work.

Scheme	Instantiation	Deniability	Assumption		Size (in bytes)	
			pre-Q	post-Q	$c$	$pk$
DH-AKEM [ABH <sup>+</sup> 21]	X25519	DR-Den*	✓	✗	32	32
ETStH-AKEM [AJKL23]	bat_257_542 + ANTRAG	✗	✗	✓	1 119	1 417
	ML-KEM-512 + Falcon-512				1 434	1 697
NIKE-AKEM [AJKL23]	SWOOSH [GdKQ <sup>+</sup> 24]	DR-Den*	✗	✓	> 221 184	> 221 184
FrodoKEX+ [CHN <sup>+</sup> 24]	n/a	DR-Den	✗	✓	72	21 300
PQ-AKEM [GJK24]	NTRU-A + GANDALF [ANTRAG, MITAKA]	HR-Den & DR-Den	✗	✓	2 044	1 664
	bat_257_542 + GANDALF [ANTRAG, MITAKA]				1 749	1 417

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	bat_257_542 + GANDALF [ANTRAG, MITAKA]				1 749	1 417
SHADOWFAX [GHJ25]	X25519 + bat_257_542 + GANDALF [ANTRAG, MITAKA]	HR-Den & DR-Den	✓	✓	1 781	1 449
	X25519 + ML-KEM-512 + GANDALF [Falcon-512]				2 076	1 729

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FrodoKEX+ [CHN <sup>+</sup> 24]	n/a	DR-Den	✗	✓	72	21 300
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	X25519 + ML-KEM-512 + GANDALF [Falcon-512]				2 076	1 729

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Deniability properties marked with an “\*” have not been formally proven in the respective work.

AKEM	Gen		Enc		Dec	
	kcc	ms	kcc	ms	kcc	ms
RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

AKEM	Gen		Enc		Dec	
	kcc	ms	kcc	ms	kcc	ms
DH-AKEM [ABH <sup>+</sup> 21]	208	0.07	628	0.21	419	0.14
RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

AKEM	Gen		Enc		Dec	
	kcc	ms	kcc	ms	kcc	ms
DH-AKEM [ABH <sup>+</sup> 21]	208	0.07	628	0.21	419	0.14
PQ-AKEM [GJK24]	24 748	8.25	1 215	0.41	332	0.11
RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

## PERFORMANCE

AKEM	Gen		Enc		Dec	
	kcc	ms	kcc	ms	kcc	ms
DH-AKEM [ABH <sup>+</sup> 21]	208	0.07	628	0.21	419	0.14
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SHADOWFAX [GHJ25]	24 770	8.27	1 846	0.62	746	0.25
RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

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SHADOWFAX [GHJ25]	24 770	8.27	1 846	0.62	746	0.25
RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms
RAPTOR [LAZ19]	71 420	23.81	7 980	2.66	505	0.17

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

## PERFORMANCE

AKEM	Gen		Enc		Dec	
	kcc	ms	kcc	ms	kcc	ms
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RSig	Gen		Sgn		Ver	
	kcc	ms	kcc	ms	kcc	ms
RAPTOR [LAZ19]	71 420	23.81	7 980	2.66	505	0.17
GANDALF [GJK24]	13 145	4.38	1 110	0.37	95	0.03

Cycle counts (in kilocycles) and time (in milliseconds) on a 3GHz Firestorm core of an Apple M1 Pro.

# SUMMARY

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- ▶ Generic combiners may be too restrictive for PQC migration.
- ▶ SHADOWFAX: a deniable AKEM combiner from NIKE, KEM and RSig:
  - ▶ Compact instantiation with  $|c| = 1.8$  KB and  $|pk| = 1.5$  KB.
- ▶ Implementation of SHADOWFAX and GANDALF ring signature scheme [GJK24].



[ia.cr/2025/154](https://ia.cr/2025/154) — [github.com/vincentvbh/shadowfax](https://github.com/vincentvbh/shadowfax)



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[p4i11ip](https://twitter.com/p4i11ip)



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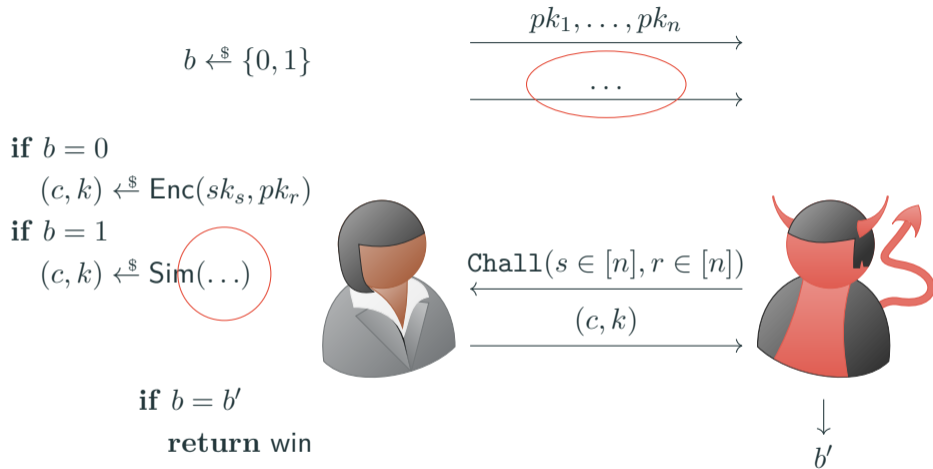
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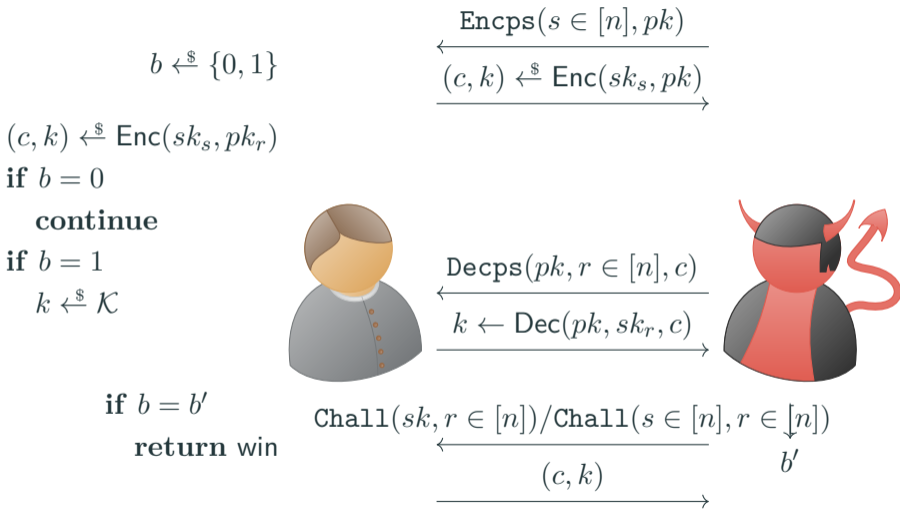
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Honest receiver:  $\text{Sim}(\emptyset), A(sk_s)$

Dishonest receiver:  $\text{Sim}(sk_r), A(sk_s, sk_r)$



## ADDITIONAL SLIDES

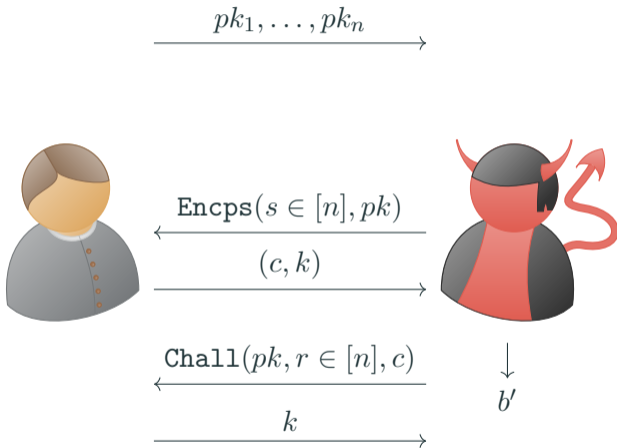
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<p><u>Games <math>(\Pi, Q_{\text{Enc}}, Q_{\text{Ch1}})</math>-Out-Aut<sub>AKEM</sub>(A)</u></p> <p><u><math>(\Pi, Q_{\text{Enc}}, Q_{\text{Dec}}, Q_{\text{Ch1}})</math>-Ins-Aut<sub>AKEM</sub>(A)</u></p> <p>for <math>i \in [n]</math></p> <p style="padding-left: 2em;"><math>(sk_i, pk_i) \leftarrow^{\\$} \text{Gen}</math></p> <p><math>\mathcal{D} \leftarrow \emptyset</math></p> <p><math>b \leftarrow^{\\$} \{0, 1\}</math></p> <p><math>b' \leftarrow^{\\$} \text{A}^{\text{Encps, Chall}}(pk_1, \dots, pk_n)</math> // Out-Aut</p> <p><math>b' \leftarrow^{\\$} \text{A}^{\text{Encps, Decps, Chall}}(pk_1, \dots, pk_n)</math> // Ins-Aut</p> <p>return <math>\llbracket b = b' \rrbracket</math></p> <p><u>Oracle Chall(<math>pk, r \in [n], c</math>)</u> // Out-Aut<sup>1</sup>.</p> <p>if <math>\exists k : (pk, pk_r, c, k) \in \mathcal{D}</math></p> <p style="padding-left: 2em;">return <math>k</math></p> <p><math>k \leftarrow \text{Dec}(pk, sk_r, c)</math></p> <p>if <math>b = 0</math></p> <p style="padding-left: 2em;">continue</p> <p>if <math>b = 1 \wedge pk \in \{pk_1, \dots, pk_n\} \wedge k \neq \perp</math></p> <p style="padding-left: 2em;"><math>k \leftarrow^{\\$} \mathcal{K}</math></p> <p style="padding-left: 2em;"><math>\mathcal{D} \leftarrow \mathcal{D} \cup \{(pk, pk_r, c, k)\}</math></p> <p>return <math>k</math></p>	<p><u>Oracle Encps(<math>s \in [n], pk</math>)</u></p> <p><math>(c, k) \leftarrow^{\\$} \text{Enc}(sk_s, pk)</math></p> <p><math>\mathcal{D} \leftarrow \mathcal{D} \cup \{(pk_s, pk, c, k)\}</math></p> <p>return <math>(c, k)</math></p> <p><u>Oracle Decps(<math>pk, r \in [n], c</math>)</u> // Ins-Aut</p> <p><math>k \leftarrow \text{Dec}(pk, sk_r, c)</math></p> <p>return <math>k</math></p> <p><u>Oracle Chall(<math>s \in [n], sk, c</math>)</u> // Ins-Aut</p> <p>if <math>\exists k : (pk_s, \mu(sk), c, k) \in \mathcal{D}</math></p> <p style="padding-left: 2em;">return <math>k</math></p> <p><math>k \leftarrow \text{Dec}(pk_s, sk, c)</math></p> <p>if <math>b = 0</math></p> <p style="padding-left: 2em;">continue</p> <p>if <math>b = 1 \wedge k \neq \perp</math></p> <p style="padding-left: 2em;"><math>k \leftarrow^{\\$} \mathcal{K}</math></p> <p style="padding-left: 2em;"><math>\mathcal{D} \leftarrow \mathcal{D} \cup \{(pk_s, \mu(sk), c, k)\}</math></p> <p>return <math>k</math></p>
--	---

```

     $b \xleftarrow{\$} \{0, 1\}$ 
if  $\exists$  previous query
    return previous  $k$ 
 $k \leftarrow \text{Dec}(pk, sk_r, c)$ 
if  $b = 1 \wedge pk$  honest  $\wedge k \neq \perp$ 
     $k \xleftarrow{\$} \mathcal{K}$ 
return  $k$ 

if  $b = b'$ 
    return win
  
```



		Honest Receiver		Dishonest Receiver	
		$sk_r$ does not leak	$sk_r$ leaks	$sk_r$ does not leak	$sk_r$ leaks
Honest Sender	$sk_s$ does not leak	$\text{Sim}(\emptyset), A(\emptyset)$	$\text{Sim}(\emptyset), A(sk_r)$	$\text{Sim}(sk_r), A(\emptyset)$	$\text{Sim}(sk_r), A(sk_r)$
	$sk_s$ leaks	$\text{Sim}(\emptyset), A(sk_s)$	$\text{Sim}(\emptyset), A(sk_s, sk_r)$	$\text{Sim}(sk_r), A(sk_s)$	$\text{Sim}(sk_r), A(sk_s, sk_r)$

$\Rightarrow$

- E.g. A non-interactive key exchange (NIKE) used for implicit authentication is dishonest receiver deniable, but not honest receiver deniable.<sup>1</sup>

<sup>1</sup>A single NIKE does not suffice as an AKEM.

Games  $(n, Q_{\text{Ch1}})$ -DR-Den<sub>AKEM, Sim</sub>(A)

$(n, Q_{\text{Ch1}})$ -HR-Den<sub>AKEM, Sim</sub>(A)

$\mathcal{R}, \mathcal{C} \leftarrow \emptyset$

**for**  $i \in [n]$

$(sk_i, pk_i) \leftarrow^{\$} \text{Gen}$

$b \leftarrow^{\$} \{0, 1\}$

$b' \leftarrow \mathbf{A}^{\text{Rev, Chall}}(pk_1, \dots, pk_n)$

**if**  $\mathcal{R} \cap \mathcal{C} \neq \emptyset$

// HR-Den

**return**  $b \leftarrow^{\$} \{0, 1\}$

// HR-Den

**return**  $\llbracket b = b' \rrbracket$

Rev( $i \in [n]$ )

$\mathcal{R} \leftarrow \mathcal{R} \cup \{i\}$

**return**  $sk_i$

Oracle Chall( $s \in [n], r \in [n]$ )

$\mathcal{C} \leftarrow \mathcal{C} \cup \{r\}$

$(c, k) \leftarrow^{\$} \text{Enc}(sk_s, pk_r)$

**if**  $b = 0$

**continue**

**if**  $b = 1$

$(c, k) \leftarrow^{\$} \text{Sim}(pk_s, pk_r, sk_r)$  // DR-Den

$(c, k) \leftarrow^{\$} \text{Sim}(pk_s, pk_r)$  // HR-Den

**return**  $(c, k)$