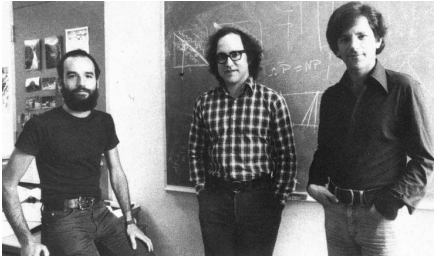
A Closer Look at Falcon

Pierre-Alain Fouque Hubert de Groote Eike Kiltz
Phillip Gajland Jonas Janneck

November 26, 2025

MOTIVATION: RSA, DH, SHOR AND QUANTUM



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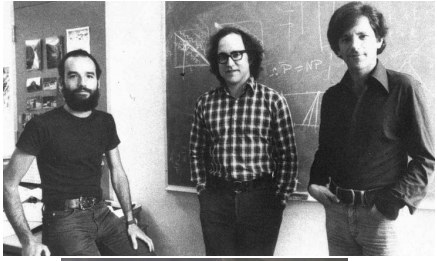

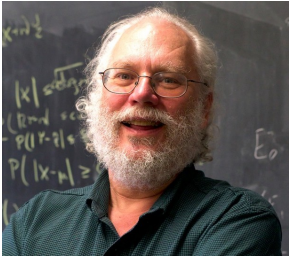

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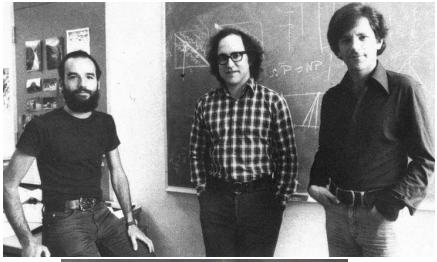

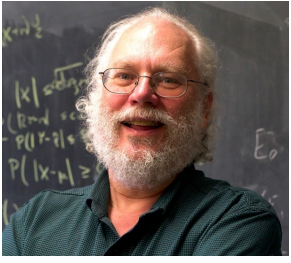

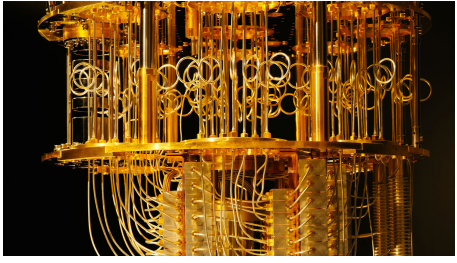

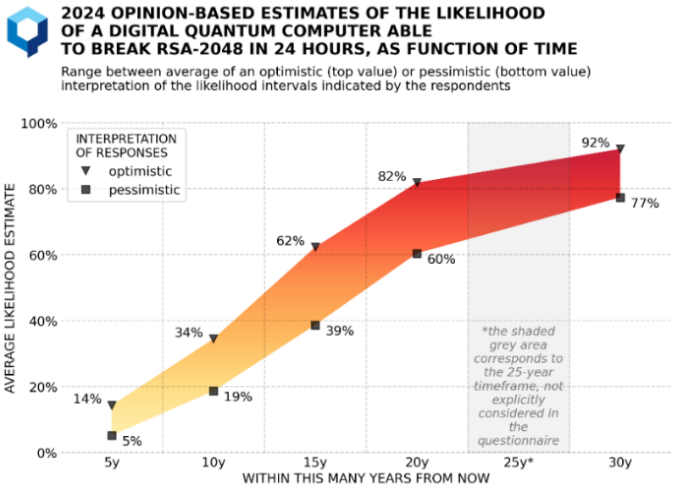
2024 OPINION-BASED ESTIMATES OF THE LIKELIHOOD OF A DIGITAL QUANTUM COMPUTER ABLE TO BREAK RSA-2048 IN 24 HOURS, AS FUNCTION OF TIME
Range between average of an optimistic (top value) or pessimistic (bottom value) interpretation of the likelihood intervals indicated by the respondents
INTERPRETATION OF RESPONSES
optimistic
pessimistic
AVERAGE LIKELIHOOD ESTIMATE
100%
80%
60%
40%
20%
0%
14%
5%
34%
19%
62%
39%
82%
60%
92%
77%
*the shaded grey area corresponds to the 25-year timeframe, not explicitly considered in the questionnaire
5y
10y
15y
20y
25y*
30y
WITHIN THIS MANY YEARS FROM NOW

- Multi-year effort to standardise post-quantum cryptography
- After 4 rounds, 5 algorithms were selected for standardisation: 2 KEM, 3 Sig

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Key Encapsulation Mechanism (KEM)	Signature Scheme (Sig)
`CRYSTALS-Kyber` [SAB+22] (`ML-KEM`)	`CRYSTALS-Dilithium` [LDK+22] (`ML-DSA`)
`HQC` [GMA+25]	`Falcon` [PFH+22] (`FN-DSA`)
	`SPHINCS+` [HBD+22] (`SLH-DSA`)

- Falcon is compact: small $|\sigma| + |pk|$

- Complicated implementation due to Gaussian Sampling
- Concrete Security?

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Signature Scheme	NIST Level	$\lvert\sigma\rvert$	$\lvert pk\rvert$	$\lvert sk\rvert$	Security	Assumptions
CRYSTALS-Dilithium	II	2 420	1 312	32	**SUF-CMA**	**MLWE, SelfTargetMSIS, MSIS**
	III	3 293	1 952	32		
	V	4 595	2 592	32		
Falcon	I	666	897	1 281	**SUF-CMA**	**second-preimage multi-target NTRU-ISIS**
	V	1 280	1 793	2 305		
SPHINCS$^+$	I	7 856	32	64	**UF-CMA**	**PRF, ITSR, SM-TCR, SM-DSPR**
	III	16 224	48	96		
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- FALCON is based on [GPV08], which proves lattice-based *full domain hash*
- Security proof is not sufficient for FALCON
  - FALCON is defined over NTRU lattices, not plain lattices
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Can Falcon be proven secure?
If so, what is its concrete security?

- Adaptation of [GPV08] uniformity results to Rényi divergence
- Proof for a modified generalisation of Falcon
  - Modification: Minor efficiency overhead due to salt resampling
  - Generalisation: Parametrised by Trapdoor Generation and Preimage Sampler
- Optimisation of the security bound and concrete bit security

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Scheme	**Notion**	**Bit Security**
Falcon$^+$-512 ($Q_s = 2^{64}$)	**UF-CMA** (Th. 1) **SUF-CMA** (Th. 2)	113
Falcon$^+$-512 ($Q_s = 2^{58}$)		119
Falcon$^+$-1024 ($Q_s = 2^{64}$)		256

Table: Provable bit security (Q_s = maximal signing queries).

BACKGROUND AND FALCON

Rényi divergence [Rén61]

- Describes closeness of two distributions (generalisation of Kullback-Leibler)
- Used for better security bounds/parameters [LSS14, BLL+15, Pre17]
- Rényi divergence: $r = R_a(\mathcal{P} \mid\mid \mathcal{Q}) = 1 + \delta$ for $\delta \geq 0$
- Applied multiplicatively
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$$(\boldsymbol{s}_1, \boldsymbol{s}_2) \mapsto \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$$

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$\underline{\mathsf{Gen}}$

$(\boldsymbol{B}, \boldsymbol{h}) \xleftarrow{\$} \mathsf{TpdGen}(\mathcal{R}, \alpha, q)$

$(sk, pk) \coloneqq (\boldsymbol{B}, \boldsymbol{h}) \in \mathbb{Z}^{2n \times 2n} \times \mathcal{R}_q$

return (sk, pk)

$\underline{\mathsf{Sgn}(sk, m)}$

$r \xleftarrow{\$} \{0,1\}^k$

$\boldsymbol{c} \coloneqq \mathsf{H}(pk, r, m) \in \mathcal{R}_q$

repeat

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$

until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$

$\sigma \coloneqq (r, \boldsymbol{s}_2) \in \{0,1\}^k \times \mathcal{R}$

return σ

$\underline{\mathsf{Ver}(pk = \boldsymbol{h}, m, \sigma = (r, \boldsymbol{s}_2))}$

$\boldsymbol{c} \coloneqq \mathsf{H}(pk, r, m)$

$\boldsymbol{s}_1 \coloneqq \boldsymbol{c} - \boldsymbol{s}_2 * \boldsymbol{h} \mod q$

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Sgn$^+(sk, m)$

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PROOF IDEA OF [GPV08] FROM ONE-WAYNESS OF $f_{\boldsymbol{h}}$ [1]

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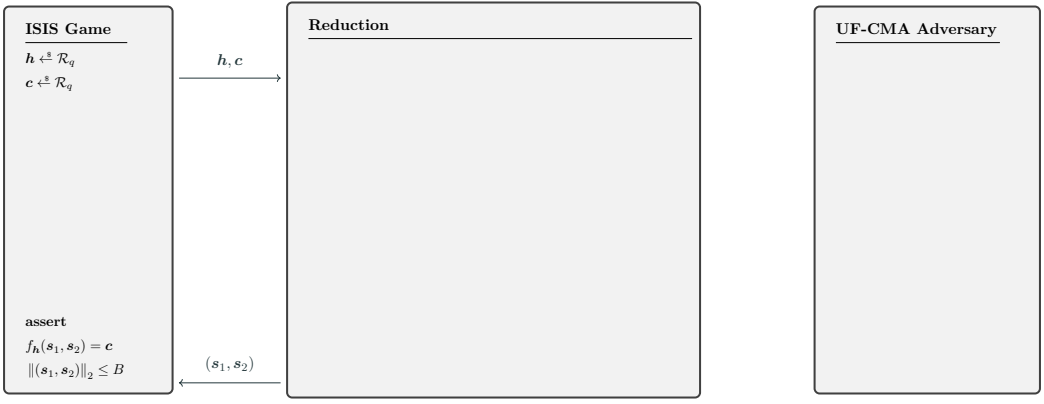

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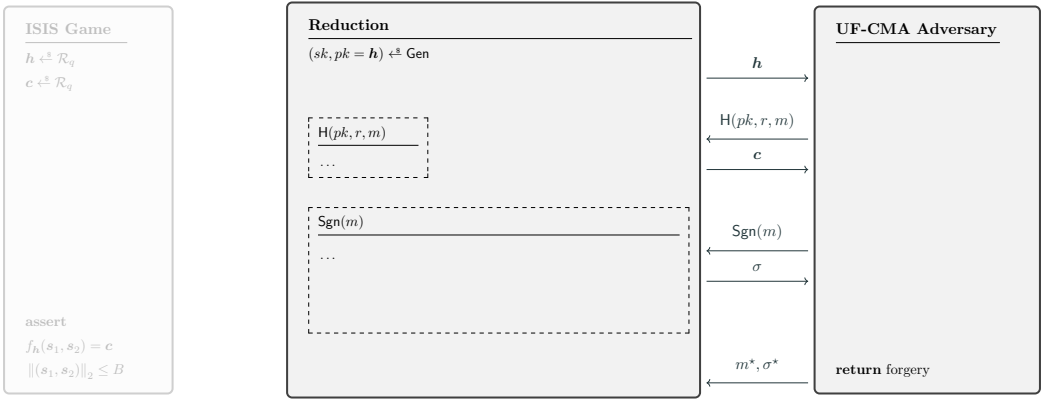

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ISIS Game

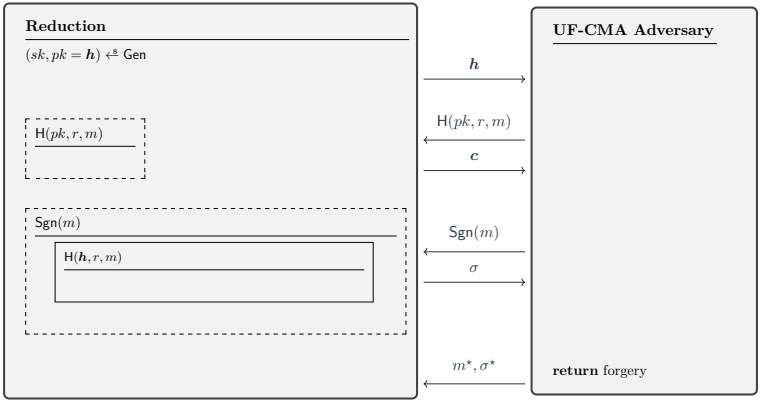
$h \xleftarrow{\$} \mathcal{R}_q$

$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$

assert

$f_h(s_1, s_2) = c$

$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$



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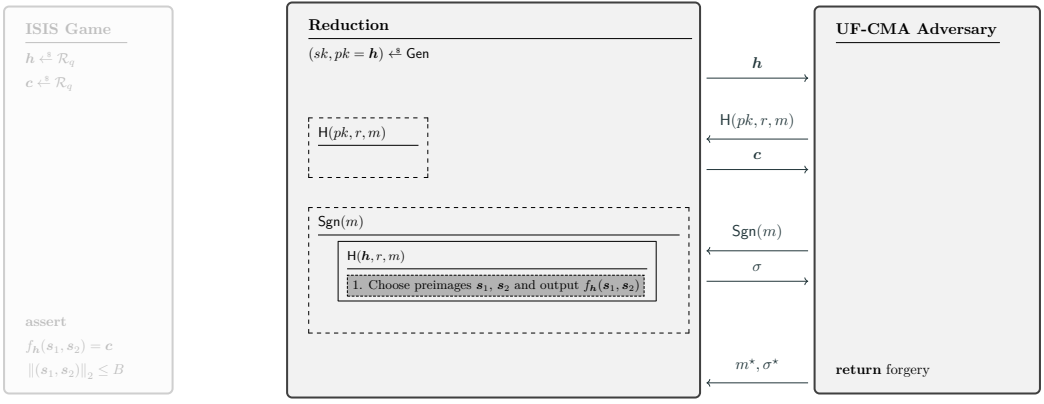

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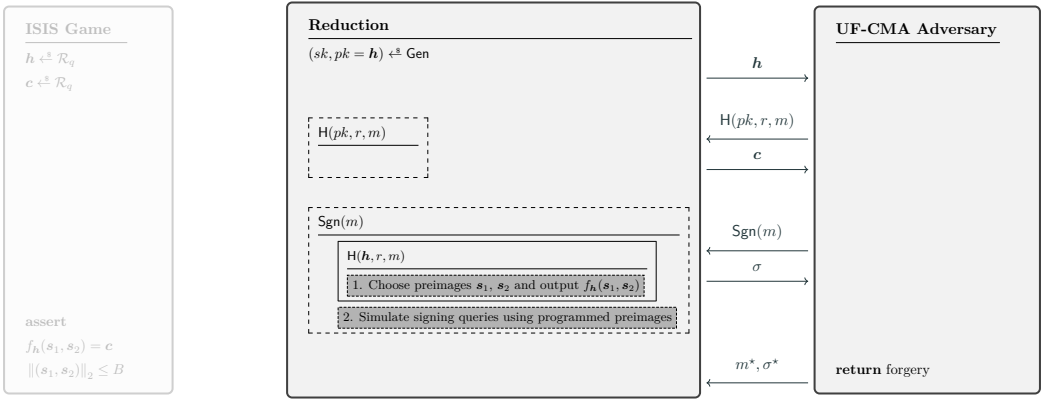

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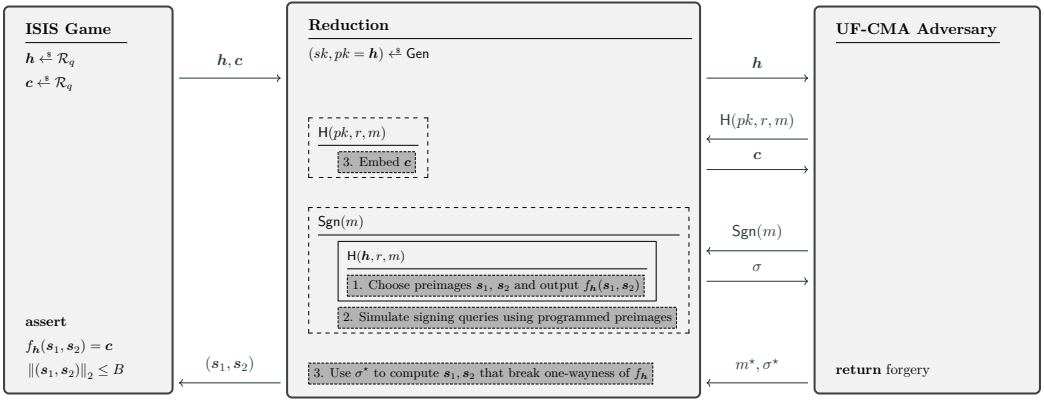
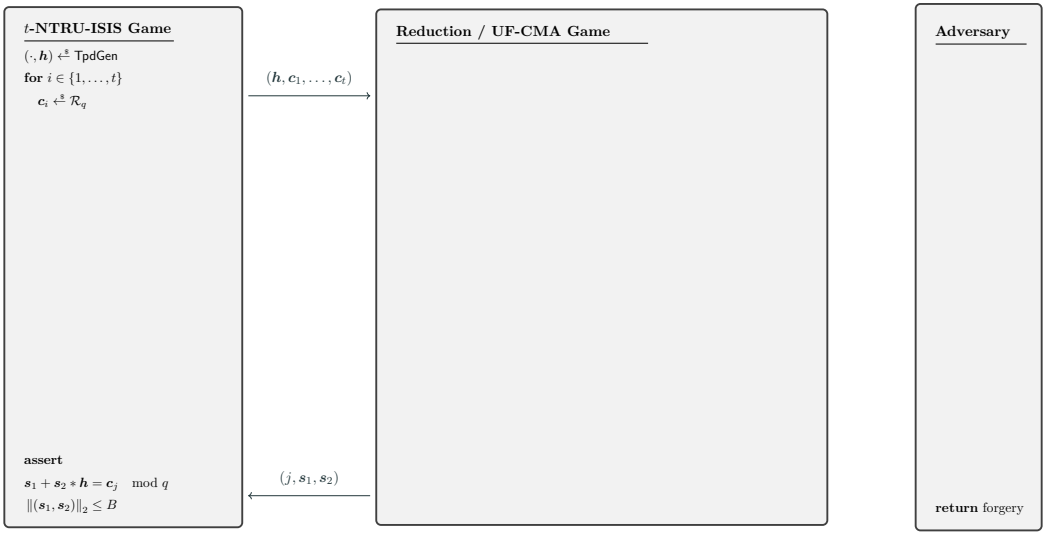
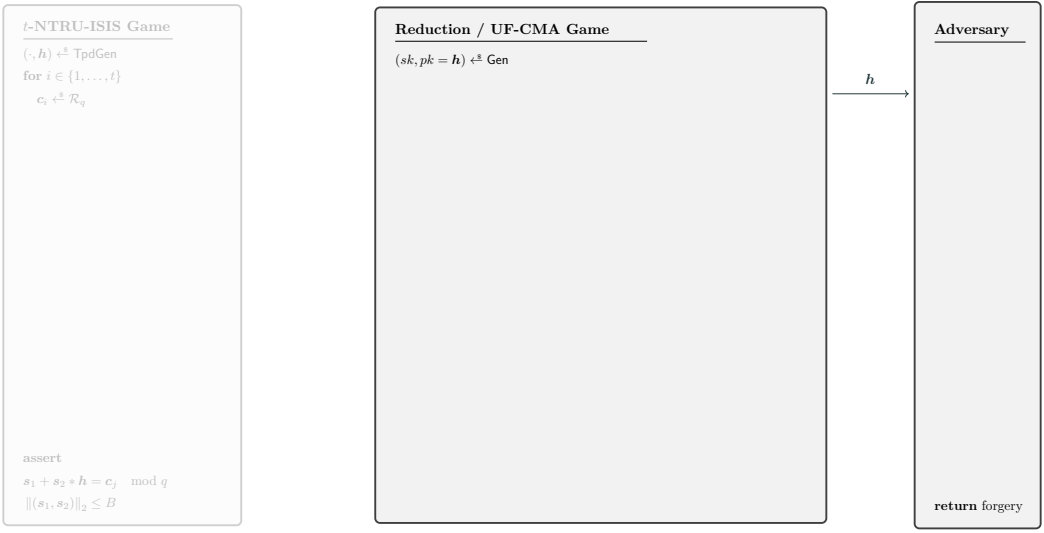
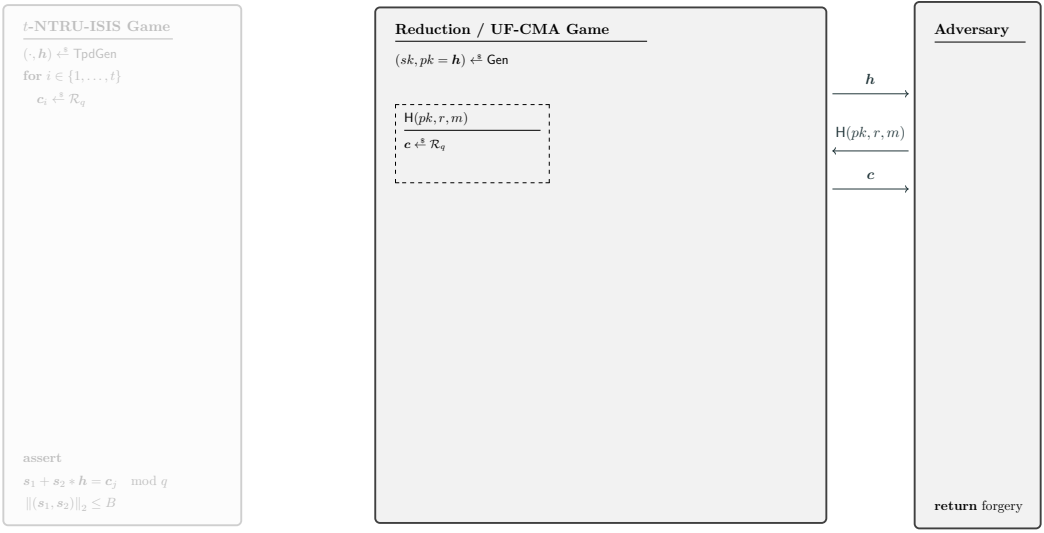


[1] We stick to the NTRU notation but [GPV08] uses plain (unstructured) lattices

PROOF


t-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}) \xleftarrow{s} \mathsf{TpdGen}$
for $i \in \{1, \ldots, t\}$
$\boldsymbol{c}_i \xleftarrow{s} \mathcal{R}_q$
$(\boldsymbol{h}, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_t)$
Reduction / UF-CMA Game
Adversary
assert
$\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} = \boldsymbol{c}_j \mod q$
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
$(j, \boldsymbol{s}_1, \boldsymbol{s}_2)$
return forgery


t-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}) \xleftarrow{s}$ TpdGen
for $i \in \{1, \ldots, t\}$
$\boldsymbol{c}_i \xleftarrow{s} \mathcal{R}_q$
assert
$\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} = \boldsymbol{c}_j \mod q$
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{s}$ Gen
$\boldsymbol{h}$
Adversary
return forgery


t-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}) \xleftarrow{s}$ TpdGen
for $i \in \{1, \ldots, t\}$
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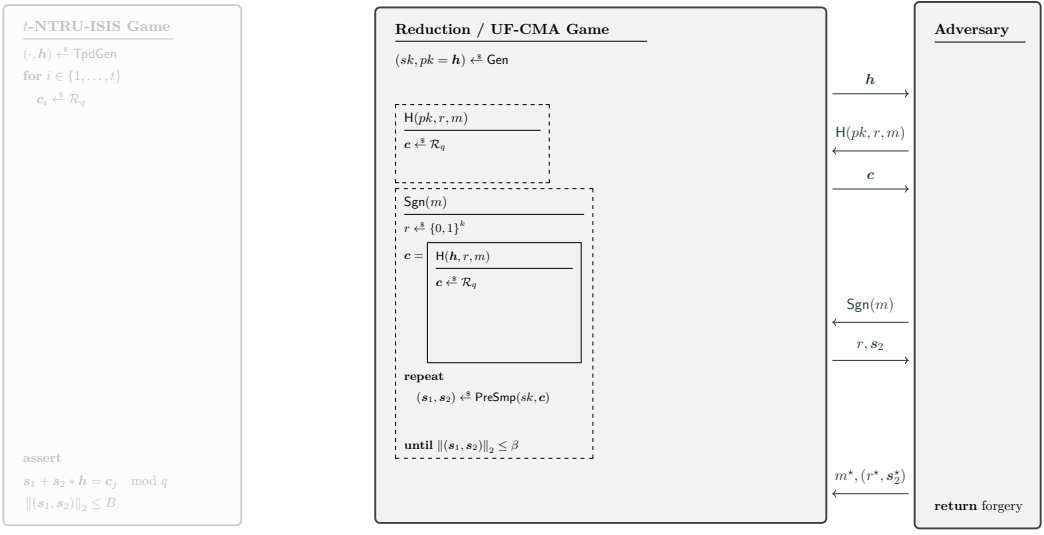
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{s}$ Gen
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{s} \mathcal{R}_q$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
Adversary
return forgery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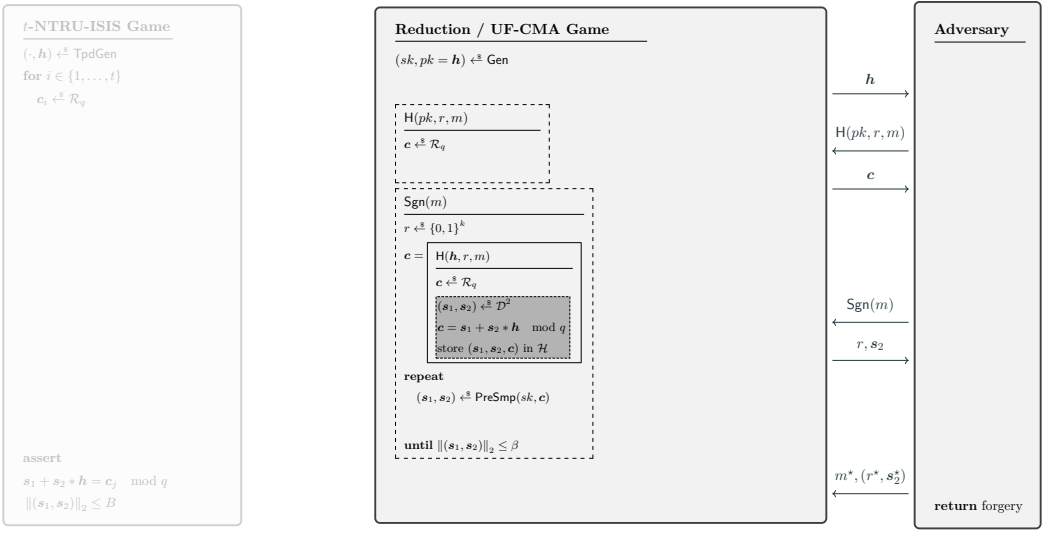
$(\cdot, \boldsymbol{h}) \xleftarrow{\$} \mathsf{TpdGen}$
for $i \in \{1, \ldots, t\}$
$\boldsymbol{c}_i \xleftarrow{\$} \mathcal{R}_q$
assert
$\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} = \boldsymbol{c}_j \mod q$
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\mathsf{Sgn}(m)$
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
repeat
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
$r, \boldsymbol{s}_2$
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery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Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\mathsf{Sgn}(m)$
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
repeat
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
$r, \boldsymbol{s}_2$
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery

- Is the distribution of $\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$ for $(\boldsymbol{s}_1, \boldsymbol{s}_2) \sim \mathcal{D}^2$ close to uniform?
- What does close mean?
- Statistical distance: **No** ($\approx 2^{-35}$)
- Rényi divergence: **Yes** (actually "maybe")

$\mathsf{H}(pk, r, m)$

$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$

$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$

store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$

- Is the distribution of $\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$ for $(\boldsymbol{s}_1, \boldsymbol{s}_2) \sim \mathcal{D}^2$ close to uniform?
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$\mathsf{H}(pk, r, m)$

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$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$

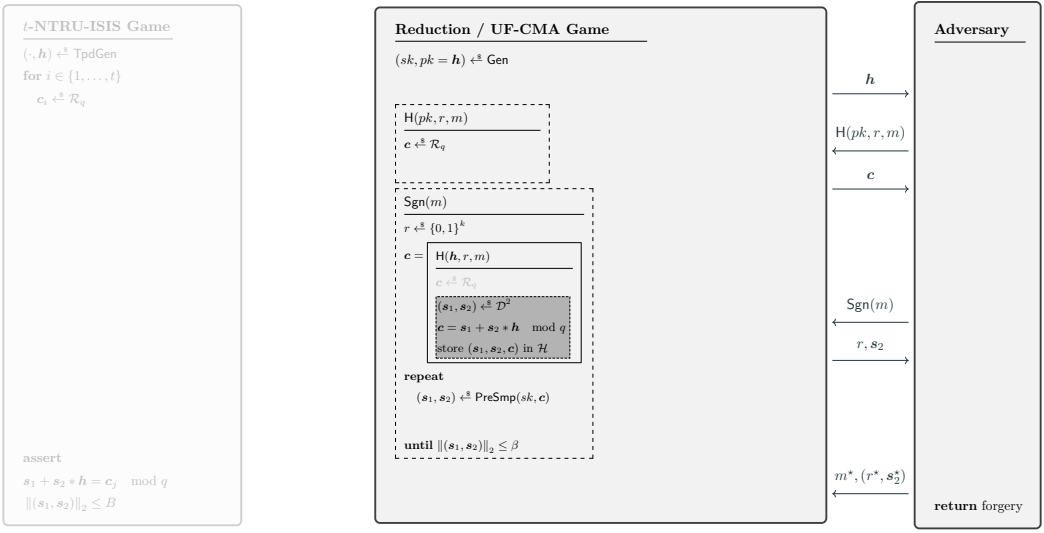
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$

store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$

Corollary: Rényi Uniformity for NTRU

Let q be prime, $\boldsymbol{h} \in \mathcal{R}_q \setminus \{\boldsymbol{0}\}$, $a \in (1, \infty)$, $\epsilon \in (0, \frac{1}{2})$, $s \geq \eta_\epsilon(\boldsymbol{\Lambda}_{\boldsymbol{h},q})$, $\mathcal{P} = \mathcal{U}(\mathcal{R}_q)$, and $\mathcal{Q}$ the distribution of $\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$ where $\boldsymbol{s}_1, \boldsymbol{s}_2 \sim \mathcal{D}_{\mathcal{R},s}$. Then,

$$R_a(\mathcal{P} \,||\, \mathcal{Q}) \lesssim 1 + \frac{2a\epsilon^2}{(1-\epsilon)^2}.$$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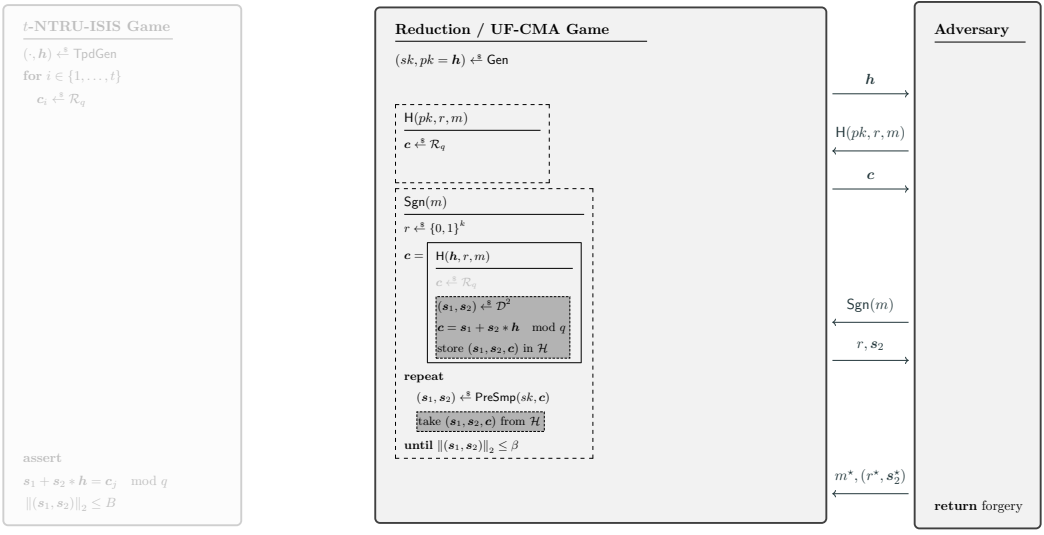
assert
$\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} = \boldsymbol{c}_j \mod q$
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\mathsf{Sgn}(m)$
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
repeat
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
$r, \boldsymbol{s}_2$
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery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store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
repeat
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
$r, \boldsymbol{s}_2$
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery

- $(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$ must be Rényi close to conditional Gaussian [Kle00, Pre17]
- Is it short enough: $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$?
- For FALCON, probability is $\approx 1 - 2^{-14}$
- Program RO with a conditional (on being small) Gaussian? Uniformity Corollary does not apply anymore!
- Salt to the rescue! $\rightarrow$ Get new preimage by sampling new salt

$\mathsf{Sgn}(m)$

$r \xleftarrow{\$} \{0,1\}^k$

$\boldsymbol{c} = \mathsf{H}(\boldsymbol{h}, r, m)$

repeat

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$

take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$

- $(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$ must be Rényi close to conditional Gaussian [Kle00, Pre17]
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take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

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take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

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$\mathsf{Sgn}(m)$

$r \xleftarrow{\$} \{0,1\}^k$

$\boldsymbol{c} = \mathsf{H}(\boldsymbol{h}, r, m)$

repeat

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$

take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$

- $(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$ must be Rényi close to conditional Gaussian [Kle00, Pre17]
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$\mathsf{Sgn}(m)$

$r \xleftarrow{\$} \{0,1\}^k$

$\boldsymbol{c} = \mathsf{H}(\boldsymbol{h}, r, m)$

repeat

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$

take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$

$\mathsf{Sgn}^+(m)$

repeat

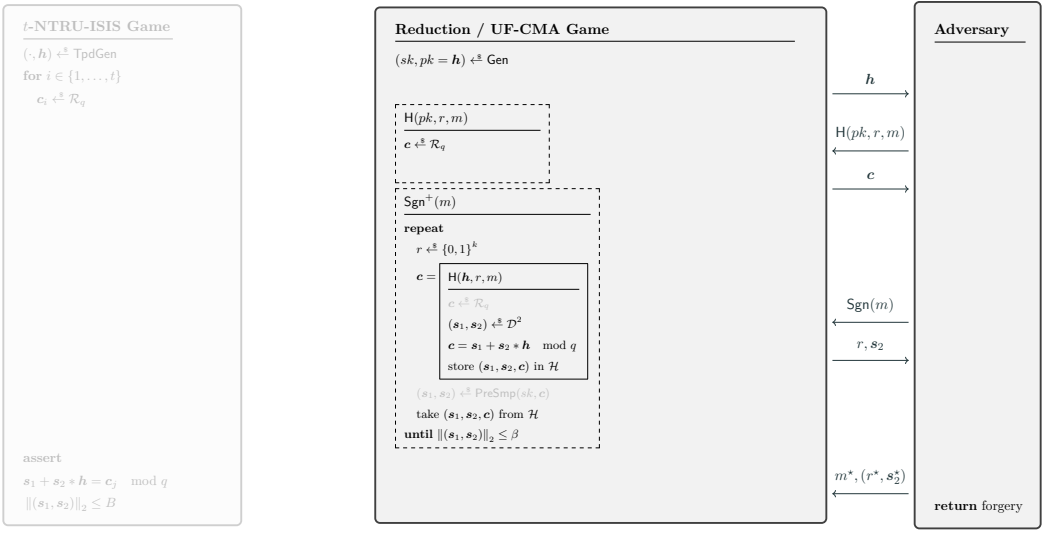
$r \xleftarrow{\$} \{0,1\}^k$

$\boldsymbol{c} = \mathsf{H}(\boldsymbol{h}, r, m)$

$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$

take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$

until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$


t-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}) \xleftarrow{\$} \mathsf{TpdGen}$
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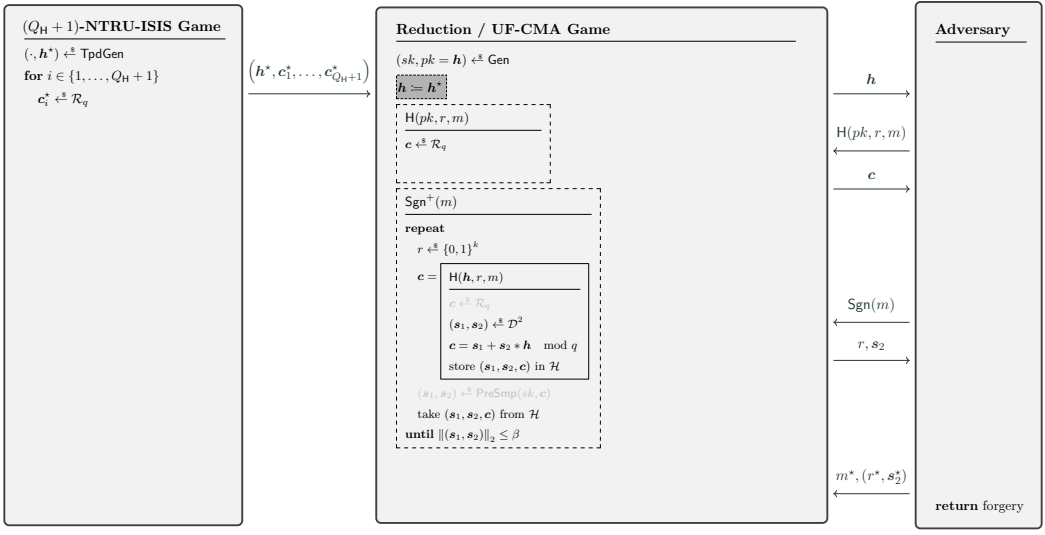
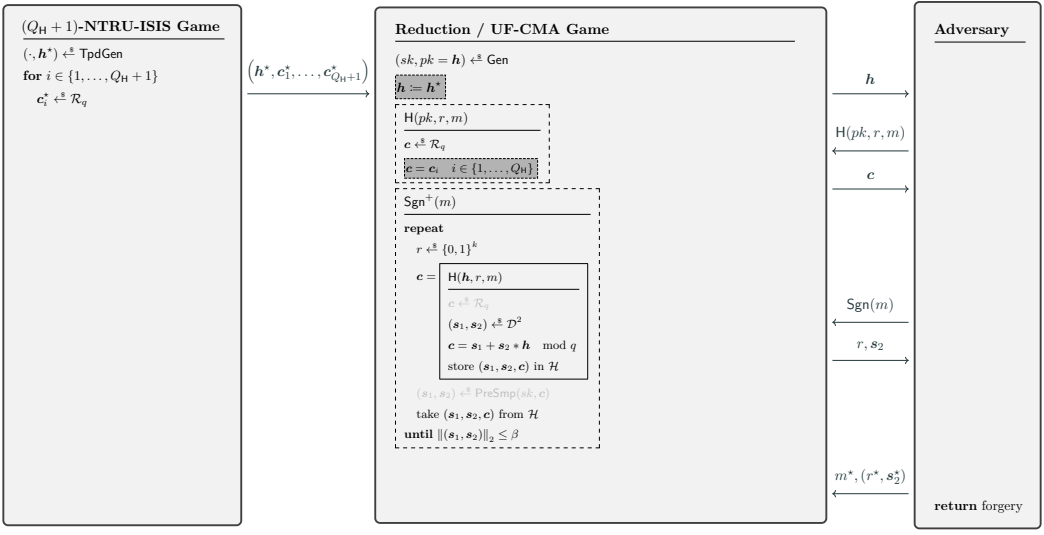
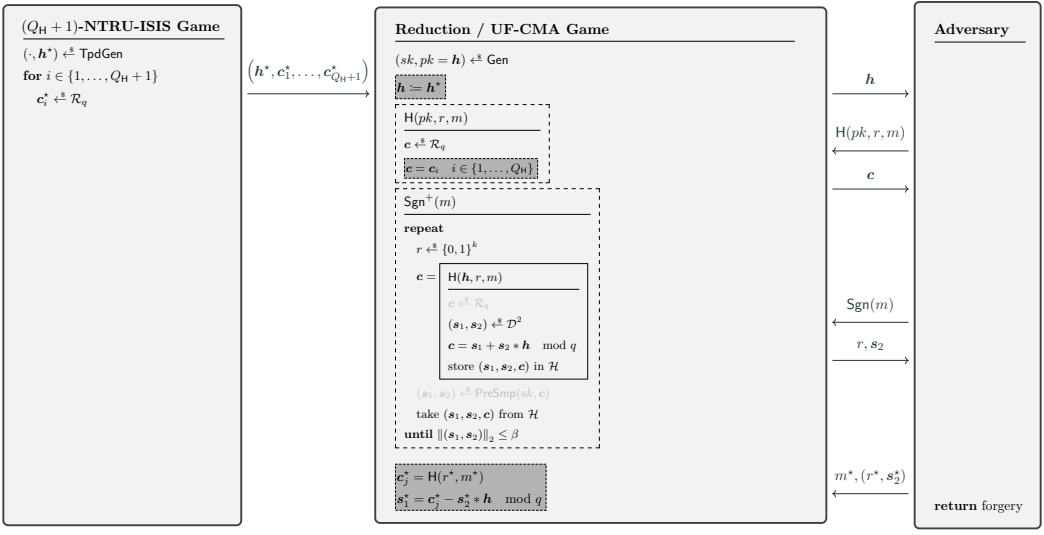
$\boldsymbol{c}_i \xleftarrow{\$} \mathcal{R}_q$
assert
$\boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} = \boldsymbol{c}_j \mod q$
$\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq B$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\mathsf{Sgn}^{+}(m)$
repeat
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
$(s_1, s_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, c)$
take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
r, s_2
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery


$(Q_{\mathsf{H}}+1)$-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}^\star) \xleftarrow{\$} \mathsf{TpdGen}$
for $i \in \{1, \ldots, Q_{\mathsf{H}}+1\}$
$\boldsymbol{c}_i^\star \xleftarrow{\$} \mathcal{R}_q$
$\left(\boldsymbol{h}^\star, \boldsymbol{c}_1^\star, \ldots, \boldsymbol{c}_{Q_{\mathsf{H}}+1}^\star\right)$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\boldsymbol{h} := \boldsymbol{h}^\star$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\mathsf{Sgn}^+(m)$
repeat
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
r, s_2
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery


$(Q_{\mathsf{H}}+1)$-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}^\star) \xleftarrow{\$} \mathsf{TpdGen}$
for $i \in \{1, \ldots, Q_{\mathsf{H}}+1\}$
$\boldsymbol{c}_i^\star \xleftarrow{\$} \mathcal{R}_q$
$\left(\boldsymbol{h}^\star, \boldsymbol{c}_1^\star, \ldots, \boldsymbol{c}_{Q_{\mathsf{H}}+1}^\star\right)$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\boldsymbol{h} := \boldsymbol{h}^\star$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\boldsymbol{c} = \boldsymbol{c}_i \quad i \in \{1, \ldots, Q_{\mathsf{H}}\}$
$\mathsf{Sgn}^+(m)$
repeat
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$
until $\|(\boldsymbol{s}_1, \boldsymbol{s}_2)\|_2 \leq \beta$
$\boldsymbol{h}$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c}$
$\mathsf{Sgn}(m)$
r, s_2
$m^\star, (r^\star, \boldsymbol{s}_2^\star)$
Adversary
return forgery


$(Q_{\mathsf{H}}+1)$-NTRU-ISIS Game
$(\cdot, \boldsymbol{h}^\star) \xleftarrow{\$} \mathsf{TpdGen}$
for $i \in \{1, \ldots, Q_{\mathsf{H}}+1\}$
$\boldsymbol{c}_i^\star \xleftarrow{\$} \mathcal{R}_q$
$\left(\boldsymbol{h}^\star, \boldsymbol{c}_1^\star, \ldots, \boldsymbol{c}_{Q_{\mathsf{H}}+1}^\star\right)$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\boldsymbol{h} := \boldsymbol{h}^\star$
$\mathsf{H}(pk, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$\boldsymbol{c} = \boldsymbol{c}_i \quad i \in \{1, \ldots, Q_{\mathsf{H}}\}$
$\mathsf{Sgn}^+(m)$
repeat
$r \xleftarrow{\$} \{0,1\}^k$
$\boldsymbol{c} =$
$\mathsf{H}(\boldsymbol{h}, r, m)$
$\boldsymbol{c} \xleftarrow{\$} \mathcal{R}_q$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathcal{D}^2$
$\boldsymbol{c} = \boldsymbol{s}_1 + \boldsymbol{s}_2 * \boldsymbol{h} \mod q$
store $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ in $\mathcal{H}$
$(\boldsymbol{s}_1, \boldsymbol{s}_2) \xleftarrow{\$} \mathsf{PreSmp}(sk, \boldsymbol{c})$
take $(\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{c})$ from $\mathcal{H}$
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$\boldsymbol{c}_j^\star = \mathsf{H}(r^\star, m^\star)$
$\boldsymbol{s}_1^\star = \boldsymbol{c}_j^\star - \boldsymbol{s}_2^\star * \boldsymbol{h} \mod q$
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Adversary
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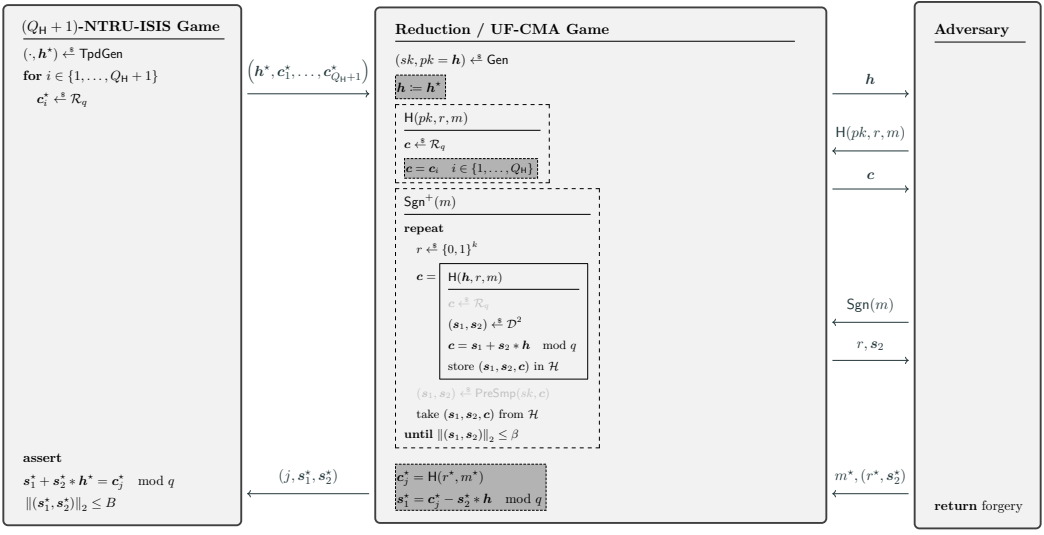
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for $i \in \{1, \ldots, Q_{\mathsf{H}}+1\}$
$\boldsymbol{c}_i^\star \xleftarrow{\$} \mathcal{R}_q$
assert
$\boldsymbol{s}_1^\star + \boldsymbol{s}_2^\star * \boldsymbol{h}^\star = \boldsymbol{c}_j^\star \mod q$
$\|(\boldsymbol{s}_1^\star, \boldsymbol{s}_2^\star)\|_2 \leq B$
$\left(\boldsymbol{h}^\star, \boldsymbol{c}_1^\star, \ldots, \boldsymbol{c}_{Q_{\mathsf{H}}+1}^\star\right)$
$(j, \boldsymbol{s}_1^\star, \boldsymbol{s}_2^\star)$
Reduction / UF-CMA Game
$(sk, pk = \boldsymbol{h}) \xleftarrow{\$} \mathsf{Gen}$
$\boldsymbol{h} := \boldsymbol{h}^\star$
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Assumption

Property of $f_{\boldsymbol{A}}$

Security Notion

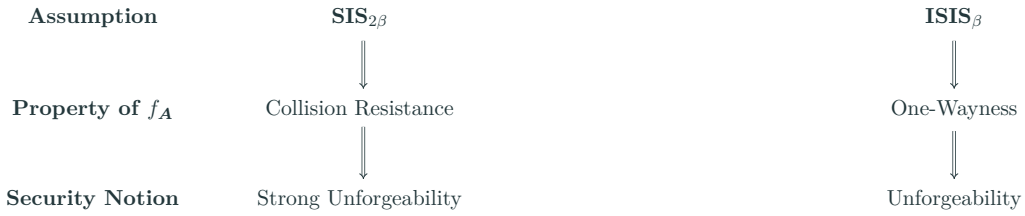
ISIS$_\beta$

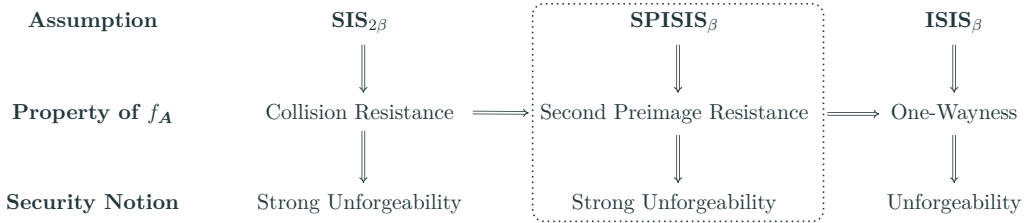
$\Downarrow$

One-Wayness

$\Downarrow$

Unforgeability


Assumption
SIS$_{2\beta}$
ISIS$_{\beta}$
Property of $f_{\boldsymbol{A}}$
Collision Resistance
One-Wayness
Security Notion
Strong Unforgeability
Unforgeability


Assumption
SIS$_{2\beta}$
SPISIS$_{\beta}$
ISIS$_{\beta}$
Property of $f_{\boldsymbol{A}}$
Collision Resistance
Second Preimage Resistance
One-Wayness
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SECURITY BOUNDS

Q_s	Maximum Signing Queries
Q_{H}	Maximum Random Oracle Queries
r_u	Rényi Loss Uniformity
r_p	Rényi Loss Preimage Sampler

$$\mathrm{Adv}^{\textbf{UF-CMA}} \leq r_u^{Q_s} \cdot r_p^{Q_s} \cdot \mathrm{Adv}^{\mathsf{Q_H}\text{-}\mathcal{R}\text{-}\textbf{ISIS}}_{B=\beta} + \ldots$$

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$$\mathrm{Adv}^{\textbf{SUF-CMA}} \leq \mathrm{Adv}^{\textbf{UF-CMA}} + r_p^{Q_s} \cdot \left(\mathrm{Adv}^{Q_s\text{-}\mathcal{R}\text{-}\textbf{SPISIS}}_{B=\beta} + \ldots\right) + \ldots$$

OLD SECURITY BOUND: STRONG UNFORGEABILITY

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$$\mathrm{Adv}^{\textbf{SUF-CMA}} \leq r_u^{Q_{\mathsf{H}}} \cdot r_p^{Q_s} \cdot \mathrm{Adv}^{\mathcal{R}\textbf{-SIS}}_{B=2\beta} + \ldots$$

- Rényi arguments rely on ϵ (error of smoothing parameter η_ϵ), with $\epsilon = 1/\sqrt{Q_s \cdot \lambda}$ and $Q_s = 2^{64}$
- $\implies$ Loss stays small for $\approx Q_s$ queries, but $r_u^{Q_{\mathsf{H}}}$ explodes for $Q_{\mathsf{H}} = 2^{96}$
- Can be solved using proof technique from [BRTZ24], but results in a significant tightness loss
- More problematic: The **SIS** bound is large. For Falcon-512, it provides only 95 bits, and for Falcon-1024, it becomes trivial since $2\beta = 16765 > 12289 = q$.

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- Optimise Rényi order a_p, a_u for Rényi terms r_p, r_u

Parameters	UF-CMA (Thm. 1)		
	FALCON$^+$-512		FALCON$^+$-1024
Bit security (core-SVP), t-$\mathcal{R}$-**ISIS**$_{q=q,\alpha=1.17,B=\beta}$	120		
Max Signing queries Q_s	2^{64}		
Max repetitions C_s	$2^{64} + 2^{50}$		
Rényi Order a_p	72.96		
Rényi Order a_u	71.73		
Bits lost from Rényi a_p	3.5		
Bits lost from Rényi a_u	3.5		
Final bit security	**113**		

- There might be better attacks against multi-target **ISIS** [2]
- Similar for **SUF-CMA**, as **UF-CMA** term dominates

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Rényi Order a_p	72.96	583.67	
Rényi Order a_u	71.73	582.46	
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	Falcon$^+$-512		Falcon$^+$-1024
Bit security (core-SVP), t-$\mathcal{R}$-**ISIS**$_{q=q,\alpha=1.17,B=\beta}$	120		278
Max Signing queries Q_s	2^{64}	2^{58}	2^{64}
Max repetitions C_s	$2^{64}+2^{50}$	$2^{58}+2^{44}$	$2^{64}+2^{36}$
Rényi Order a_p	72.96	583.67	157.05
Rényi Order a_u	71.73	582.46	155.92
Bits lost from Rényi a_p	3.5	0.5	4
Bits lost from Rényi a_u	3.5	0.5	4
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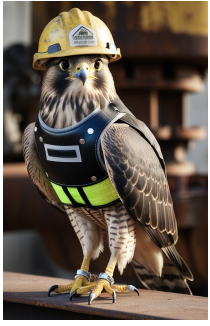
SUMMARY

- Minor, conservative modification allows for the *first concrete proof* of FALCON
- Adaptation of [GPV08] to work with Rényi divergence
- Optimisation of the security bound and concrete bit security (for FFO sampler)



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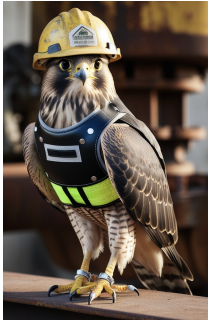

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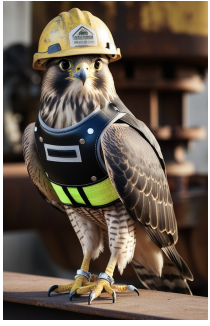

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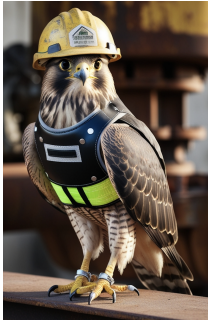

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